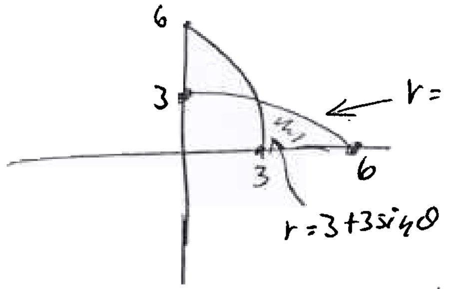
115

~~95~~

3. Sketch the region in the first quadrant that is inside $r = 3+3\cos\theta$ and outside $r = 3+3\sin\theta$, and find its area.



$= \frac{9}{2}\int_0^{\frac{\pi}{4}} 2\cos\theta - 2\sin\theta + \cos 2\theta \, d\theta$

$= \frac{9}{2}\left(2\sin\theta + 2\cos\theta + \frac{\sin 2\theta}{2}\right)\Big]_0^{\frac{\pi}{4}}$

$= \frac{9}{2}\left(2\frac{\sqrt{2}}{2} + 2\frac{\sqrt{2}}{2} + \frac{1}{2} - 2\right)$

$= \frac{9}{2}\left(2\sqrt{2} + \frac{1}{2} - 2\right)$

8

intersection $\cancel{3}+3\sin\theta = \cancel{3}+3\cos\theta$

$\frac{\sin\theta}{\cos\theta} = 1$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

$\frac{1}{2}\int_0^{\frac{\pi}{4}} (3+3\cos\theta)^2 - (3+3\sin\theta)^2 \, d\theta$

$\frac{9}{2}\int_0^{\frac{\pi}{4}} (+2\cos\theta + \cos^2\theta) - (1+2\sin\theta+\sin^2\theta)\, d\theta$

4. Let $f(x) = \dfrac{x}{e^{3x}}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find all local extreme points of $y = f(x)$ and all points of inflection of $y = f(x)$. Graph $y = f(x)$.

$f = xe^{-3x}$

$f' = -3xe^{-3x} + e^{-3x}$

$= e^{-3x}(-3x+1)$

$f'' = -3e^{-3x} - 3e^{-3x}(-3x+\ )$

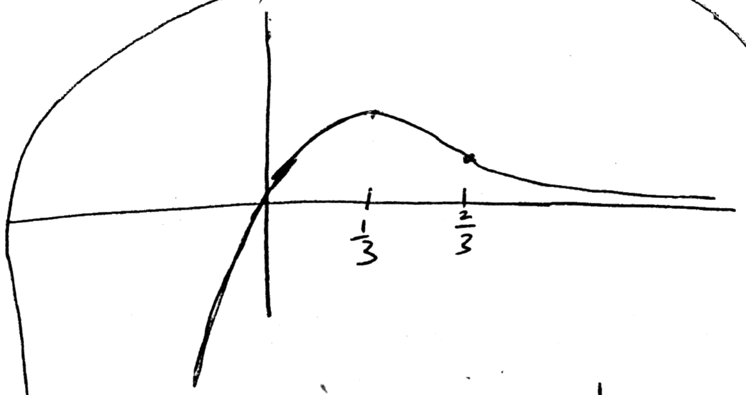
$-3e^{-3x}(-3x+2)$

f ' pos | f ' neg
$\frac{1}{3}$

8

f '' neg | f '' pos
$\frac{2}{3}$

$\lim_{x\to\infty} f = 0$



f is inc for $x < \frac{1}{3}$

f is dec for $\frac{1}{3} < x$

f is c.u for $\frac{2}{3} < x$

f is c.d. for $x < \frac{2}{3}$

$\left(\frac{1}{3}, \frac{1}{3e}\right)$ local max

no local min

$\left(\frac{2}{3}, \frac{2}{3e^2}\right)$ is p.i.