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18. Find the derivative of $y = \ln(e^{2x} + \sqrt{2x^2+3})$.

$$y' = \frac{2e^{2x} + \frac{4x}{2\sqrt{2x^2+3}}}{e^{2x} + \sqrt{2x^2+3}}$$

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$$\cos x = -\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

19. Approximate $\int_0^1 \cos(x^2)\,dx$ with an error less than 10^{-3}

$$\int_0^1 -\frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots dx$$

$$x \quad \frac{x^5}{5(2!)} + \frac{x^9}{9(4!)} - \frac{x^{13}}{13(6!)} + \cdots \Big]_0^1$$

$$= 1 \quad \frac{1}{5(2!)} + \frac{1}{9(4!)} \quad \frac{1}{13(6!)} + \frac{1}{17(8} - \cdots$$

This s an alternating series. The terms decrease to zero because $\frac{1}{(2n+1)n!} > \frac{1}{(2n+5)(n+2)!}$ and $\lim_{n\to\infty} \frac{1}{(2n+1)n!} = 0$

So AST says

$$\left|\int_0^1 \cos(x^2)\,dx - \left(1 - \frac{1}{5(2!)} + \frac{1}{9(4!)}\right)\right| < \frac{1}{13(6!)} < \frac{1}{1000}$$

$\therefore 1 - \frac{1}{5(2!)} + \frac{1}{9(4!)}$ approximates $\int_0^1 \cos(x^2)dx$ with an error less than 10^{-3}.