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16. Does $\sum_{n=2}^{\infty} \frac{2^n+3^n}{4^n}$ converge? (Explain your answer.)

$$\frac{2^n+3^n}{4^n} < \frac{3^n+3^n}{4^n} = 2\frac{3^n}{4^n}$$

$\sum 2\frac{3^n}{4^n} = 2\sum\left(\frac{3}{4}\right)^n$ converges because it is a geometric series with ratio $\frac{3}{4}<1$ so the Comparison test shows that

$\sum \frac{2^n+3^n}{4^n}$ also converges.

8

17. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 350° F and left to cool in a room at 70° F, then its temperature T after t hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T-70).$$

If the temperature fell to 250° F after one hour, what will it be after 3 hours? (You may leave "ln" in your answer.)

8

$$\frac{dT}{T-70} = k\,dt$$

$$\ln|T-70| = kt + C$$

$$T-70 = \pm e^c e^{kt}$$

$$T(0) = 350$$

$$350-70 = \pm e^c e^{k0}$$

$$280 = \pm e^c$$

$$T-70 = 280e^{kt}$$

$$T(1) = 250$$

$$250-70 = 280e^k$$

$$\frac{18\cancel{0}}{28\cancel{0}} = e^k$$

$$\ln\frac{18}{28} = k$$

$$T(t)-70 = 280e^{\ln\left(\frac{18}{28}\right)t}$$

$$T(3) = 70+280e^{3\ln\left(\frac{18}{28}\right)}$$