83 ~~84~~

7. Approximate sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ with an error of at most $\frac{1}{100}$. sure to explain what you are doing.)

The Alternating Series Test applies because this is an alternating series and the numbers $1 > \frac{1}{2^4} > \frac{1}{3^4} > \cdots$ decrease to 0. So the series converges and

$$\left| \text{The total sum} - \sum_{n=1}^{N} \frac{(-1)^{n+1}}{n^4} \right| < \frac{1}{(N+1)^4}$$

so

$$\left| \text{The total sum} - \left(\frac{1}{1} - \frac{1}{2^4} + \frac{1}{3^4}\right) \right| < \frac{1}{4^4} < \frac{1}{100}$$

So the sum of the series is within $\frac{1}{100}$ of

$$1 - \frac{1}{2^4} + \frac{1}{3^4}$$

8. A ball is dropped from the height of 10 feet. Each time it hits the floor it rebounds to $\frac{2}{3}$ its previous height. Find the total distance it travels.

$10\downarrow \quad \uparrow \frac{2}{3}(10) \quad \downarrow \frac{2}{3}(10) \quad \uparrow \left(\frac{2}{3}\right)^2 10 \quad \downarrow \left(\frac{2}{3}\right)^2 10 \quad \uparrow \left(\frac{2}{3}\right)^3 10 \quad \downarrow \left(\frac{2}{3}\right)^3 10$

$$\text{distance} = 10 + \frac{2}{3}(20) + \left(\frac{2}{3}\right)^2(20) + \left(\frac{2}{3}\right)^3 20 + \cdots$$

$$= 10 + \frac{\frac{2}{3}(20)}{1 - \frac{2}{3}} \text{ ft}$$

geometric series with

$a = \frac{2}{3}(20)$

$r = \frac{2}{3}$