

# Fall 1998 Math 142 Exam 3

PRINT Your Name: \_\_\_\_\_ Recitation Time: \_\_\_\_\_

There are 9 problems on 4 pages. Eight of problems are each worth 10 points. The other problem has two parts and each part is worth 10 points. SHOW your work.

**CIRCLE** your answer. **CHECK** your answer whenever possible.  
**NO CALCULATORS!**



1. Find  $\int_{-1}^2 \frac{1}{(x-1)^2} dx$ .



$$\lim_{b \rightarrow 1^-} \int_{-1}^b \frac{1}{(x-1)^2} dx + \lim_{a \rightarrow 1^+} \int_a^2 \frac{1}{(x-1)^2} dx = \lim_{b \rightarrow 1^-} \left[ \frac{-1}{x-1} \right]_{-1}^b + \lim_{a \rightarrow 1^+} \left[ \frac{-1}{x-1} \right]_a^2$$

$$\lim_{b \rightarrow 1^-} \left( \frac{-1}{b-1} + \frac{1}{-2} \right) + \lim_{a \rightarrow 1^+} \left( \frac{-1}{2} + \frac{1}{a-1} \right) = \infty$$

This integral diverges to  $\infty$ .

2. Find the limit of the sequence whose  $n^{\text{th}}$  term is  $a_n = (1 - \frac{1}{n})^{2n}$

$$\lim_{n \rightarrow \infty} \ln a_n = \lim_{n \rightarrow \infty} 2n \ln \left( 1 - \frac{1}{n} \right) = \lim_{n \rightarrow \infty} \frac{2 \ln \left( 1 - \frac{1}{n} \right)}{\frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{\frac{+2}{n^2}}{\frac{-1}{n^2}} = \lim_{n \rightarrow \infty} \frac{-2}{1} = -2$$

$= -2$

So  $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} e^{\ln a_n} = e^{-2}$

3. Find  $\int \frac{\ln x}{x^3} dx$

$u = \ln x \quad v = \frac{-1}{2x^2}$   
 $du = \frac{1}{x} dx \quad dv = x^{-3} dx$

$$\frac{-\ln x}{2x^2} + \frac{1}{2} \int x^{-3} dx$$

$$= \frac{-\ln x}{2x^2} - \frac{1}{4x^2} + C$$