Math 142, Exam 2, Fall 1998, Problems 5, 6, 7, and 8

5. Find $\int \ln x \, dx$.

Use integration by parts with

$$\begin{array}{ll} u = \ln x & v = x \\ du = \frac{1}{x} dx & dv = dx. \end{array}$$

We have

$$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} dx = \boxed{x \ln x - x + C.}$$

6. Find $\int \sin^2 x \cos^3 x \, dx$.

Let $u = \sin x$ **. It follows that** $du = \cos x dx$ **. We have**

$$\int \sin^2 x \cos^3 x \, dx = \int \sin^2 x (1 - \sin^2 x) \cos x \, dx = \int (u^2 - u^4) du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} = \boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5}.}$$

7. Find $\int \sin^2 x \, dx$.

We have

$$\int \sin^2 x \, dx = \frac{1}{2} \int 1 - \cos(2x) \, dx = \boxed{\frac{1}{2} \left(x - \frac{\sin(2x)}{2} \right).}$$

8. Find $\int \frac{\ln x}{x} \, dx$.

Let $u = \ln x$ **. It follows that** $du = \frac{1}{x} dx$ **, and**

$$\int \frac{\ln x}{x} \, dx = \int u \, du = \frac{u^2}{2} + C = \boxed{\frac{(\ln x)^2}{2} + C.}$$