

# 142 Exam 4 Fall 2001

PRINT Your Name: \_\_\_\_\_

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. **CIRCLE** your answer. **NO CALCULATORS!** If you want to pick up your exam before Monday, write a short note to that effect on the top of this page and I will leave your exam outside my office door, before I go home tonight.

1. Does  $\sum_{n=1}^{\infty} \frac{4}{n}$  converge? Justify your answer.

This series is 4 times the harmonic series.

This series diverges.

2. Does  $\sum_{n=1}^{\infty} \frac{n+3}{n^2\sqrt{n}}$  converge? Justify your answer.

Do a limit comparison to  $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ , which converges because it is a p-series with  $p = \frac{3}{2}$  and  $\frac{3}{2} > 1$ .

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \lim_{n \rightarrow \infty} \frac{n+3}{n^2\sqrt{n}} \cdot n^{3/2} = \lim_{n \rightarrow \infty} \frac{n+3}{n} = \lim_{n \rightarrow \infty} \left(1 + \frac{3}{n}\right) = 1, \text{ 1 is a number,}$$

$$a_n = \frac{n+3}{n^2\sqrt{n}}, \quad b_n = \frac{1}{n^{3/2}}$$

1 is not  $0$  or  $\infty$ , so both series converge or both series diverge. We

know that  $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$  converges, so

$$\sum_{n=1}^{\infty} \frac{n+3}{n^2\sqrt{n}} \text{ also converges}$$