

9. Find  $\int_1^{\infty} \frac{x}{e^x} dx = \lim_{b \rightarrow \infty} \left( -xe^{-x} + \int e^{-x} dx \right) \Big|_1^b = \lim_{b \rightarrow \infty} \left[ -xe^{-x} - e^{-x} \right]_1^b$

$u = x \quad v = -e^{-x} \quad \frac{d}{dx}(uv) = +xe^{-x} - e^{-x} + e^{-x}$   
 $du = dx \quad dv = e^{-x} dx$

$= \lim_{b \rightarrow \infty} \left[ \frac{-b}{e^b} - \frac{1}{e^b} - \left( -\frac{1}{e} - \frac{1}{e} \right) \right] = \lim_{b \rightarrow \infty} \left[ \frac{-1}{e^b} - 0 + \frac{2}{e} \right] = \frac{2}{e}$

$t, b \rightarrow \infty$

10. Consider the series  $\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$ . Find a closed formula for the partial sum

$s_n = \sum_{k=1}^n \ln\left(\frac{k}{k+1}\right)$ . (In other words, I want you to find a formula which is equal to  $s_n$ . Your formula is not allowed to contain any "dots" or any summation signs.) Does the original series converge or diverge? Find the limit of the series, if possible.

$s_n = \ln\left(\frac{1}{2}\right) + \ln\left(\frac{2}{3}\right) + \ln\left(\frac{3}{4}\right) + \dots + \ln\left(\frac{n}{n+1}\right)$

$\left(\ln 1 - \ln 2\right) + \left(\ln 2 - \ln 3\right) + \left(\ln 3 - \ln 4\right) + \dots + \left(\ln n - \ln(n+1)\right) = -\ln(n+1)$

The original series diverges to  $-\infty$