5. Find $\int \frac{6x^2-3x+1}{(4x+1)(x^2+1)}dx$. Check your answer.

$$\frac{6x^2-3x+1}{(4x+1)(x^2+1)} = \frac{A}{4x+1} + \frac{Bx+c}{x^2+1}$$

check

$-3x+1 = A(x^2+1) + (Bx+c)(4x+1)$ *

$-3x+1 = (A+4B)x^2 + (B+4c)x + (A+c)$

$\frac{2}{4x+1} + \frac{x-1}{x^2+1} = \frac{2x^2+2 + 4x^2-4x+x-1}{(4x+1)(x^2+1)}$

$= \frac{6x^2-3x-1}{(4x+1)(x^2+1)}$

Let $x = -\frac{1}{4}$ in *

$\frac{6}{16} + \frac{3}{4} + 1 = A(\frac{1}{16}+1)$

$\frac{34}{16} = \frac{6+12+16}{16} = A(\frac{17}{16})$

$\frac{16}{17} \cdot \frac{34}{16} = A$

$2 = A$

So the original problem equals

$\int \frac{2}{4x+1} + \frac{x}{x^2+1} + \frac{-1}{x^2+1} dx$

$= \boxed{\frac{1}{2}\ln|4x+1| + \frac{1}{2}\ln(x^2+1) - \tan^{-1}x + c}$

$6 = 2+4B$ So $B=1$

$1 = 2+c$ So $c=-1$

✓: $\frac{d}{dx}(PA) = \frac{1}{2}\cdot\frac{4}{4x+1} + \frac{1}{2}\cdot\frac{2x}{x^2+1} - \frac{1}{x^2+1}$

6. Consider the sequence whose n^{th} term is $a_n = \left(\frac{n-3}{n}\right)^n$ Find the limit of this sequence.

$\lim_{n\to\infty}\left(\frac{n-3}{n}\right)^n = \lim_{n\to\infty}\left(1-\frac{3}{n}\right)^n = \boxed{e^{-3}}$

because $\lim_{h\to\infty}\left(1+\frac{h}{n}\right)^n = e^h$ for all h.