3. Find the general solution of $\frac{dy}{dx} - 3y = xe^{3x}$. Check your answer.

This is a First Order Linear Differential Equation. I let $\mu = e^{\int P(x)\,dx} = e^{\int -3\,dx} = e^{-3x}$

$$e^{-3x}\frac{dy}{dx} \quad 3e^{-3x}y = x$$

$$\frac{d}{dx}\left(ye^{-3x}\right) = x$$

$$ye^{-3x} = \int x\,dx$$

$$ye^{-3x} = \frac{x^2}{2} + C$$

$$y = \frac{x^2}{2}e^{3x} + Ce^{3x}$$

V: $\frac{dy}{dx} - 3y = \frac{x^2}{2}\cancel{e^{3x}}(3) + xe^{3x} + 3C\cancel{e^{3x}} - 3\left(\cancel{\frac{x^2}{2}e^{3x}} + \cancel{Ce^{3x}}\right) = xe^{3x}$ ✓

4. Find $\int e^{-x}\cos x\,dx$. Check your answer.

$$\int e^{-x}\cos x\,dx = e^{-x}\sin x + \int e^{-x}\sin x\,dx = e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx$$

$u = e^{-x}$ $V = \sin x$ $\quad$ $u = e^{-x}$ $V = -\cos x$

$du = -e^{-x}dx$ $dV = \cos x\,dx$ $\quad$ $du = -e^{-x}$ $dV = \sin x\,dx$

so $\int e^{-x}\cos x\,dx = \frac{1}{2}\left(e^{-x}\sin x - e^{-x}\cos x\right) + C$

V: $\frac{d}{dx}(RA) = \frac{1}{2}\begin{pmatrix} e^{-x}\cos x & - e^{-x}\sin x \\ + e^{-x}\cos x & + e^{-x}\sin x \end{pmatrix}$ ✓