Fall 2001, Exam 2, Math 142, The solution to problems 7, 8, 9, 10, 11.

7. Find $\int \frac{e^{2x}}{1+e^{2x}} dx$. Check your answer.
Let $u = 1 + e^{2x}$ **. Then** $du = 2e^{2x}\, dx$ **. The original integral is equal to**

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + c = \boxed{\frac{1}{2}\ln(1+e^{2x}) + C}.$$

Of course, the derivative of $\frac{1}{2}\ln(1+e^{2x})$ **is** $\frac{e^{2x}}{1+e^{2x}}$ **.**

8. Find $\int \frac{e^{x}}{1+e^{2x}} dx$. Check your answer.
Let $u = e^x$ **. Then** $du = e^x\, dx$ **. The original integral is equal to**

$$\int \frac{du}{1+u^2} du = \arctan u + c = \boxed{\arctan e^x + C}.$$

Of course, the derivative of $\arctan e^x$ **is** $\frac{e^x}{1+e^{2x}}$ **.**

9. Solve the differential equation $\frac{dy}{dx} - \frac{y}{x} = 3x^3$. Check your answer.
This is a first order linear differential equation. Let

$$\mu = e^{\int \frac{-1}{x}\, dx} = e^{-\ln x} = \frac{1}{x}.$$

Multiply both sides by μ **to get**

$$\frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 3x^2.$$

Integrate with respect to x **to get**

$$\frac{1}{x}y = x^3 + C.$$

So, $\boxed{y = x^4 + cx}$ **. We check this answer. Observe that**

$$\frac{dy}{dx} - \frac{y}{x} = 4x^3 + c - (x^3 + c) = 3x^3.$$

10. Let $f(x) = x^2 - 2x$ for $x \le 1$. Find $f^{-1}(x)$.
Let $y = f^{-1}(x)$ **. So,** $f(y) = x$ **. It follows that** $y^2 - 2y = x$ **:**

$$y^2 - 2y + 1 = x + 1$$
$$(y-1)^2 = x + 1$$
$$y - 1 = \pm\sqrt{x+1}$$

But, y **is in the domain of** f **; so,** $y \le 1$ **. Thus,** $y = 1 - \sqrt{x+1}$ **. The answer is** $\boxed{f^{-1}(x) = 1 - \sqrt{x+1}}$ **.**

11. Find the volume of the solid generated by revolving the region bounded by $y = e^x$, the $x-$ axis, the $y-$ axis, and $x = 1$ about the x-axis.
Spin the rectangle. Get a disc of volume $\pi r^2 t$ **, where** $t = dx$ **, and** r **is equal to the** y**-coordinate, which is** e^x **. So the volume is**

$$\pi\int_0^1 e^{2x} dx = \pi \frac{e^{2x}}{2}\Big|_0^1 = \boxed{\frac{\pi}{2}(e^2 - 1)}.$$

The picture is on a separate page.