123

4. Find the Taylor polynomial of degree three, $P_3(x)$, for $f(x)$ $\ln(1 \; x)$ about $a = 0$.

$f(x) = \ln(1-x)$ $\quad f(0)=0$

$f'(x) = \frac{-1}{1-x}$ $\quad f'(0) = -1$

$f''(x) = \frac{-}{(1-x)^2}$ $\quad f''(0) = -1$

$f'''(x) = \frac{2}{(1-x)^3}$ $\quad f'''(0) = -2$

$f^{(4)}(x) \quad \frac{6}{(1-x)^4}$

$$P_3(x) = -x - \frac{x^2}{2} - \frac{x^3}{3}$$

6

5. Find an upper bound for the difference between $f(x)$ and $P_3(x)$ (from problem 4) when $|x| \le \frac{1}{100}$.

$$|f(x) - P_3(x)| = |R_3(x)| = \left| \frac{x}{4!} \right| = \frac{}{(1-c)^4 \, 4!} \le \frac{1}{4(.99)^4 (100)^4}$$

$$-\frac{1}{100} \le c \le \frac{1}{100}$$

$$.99 = 1 - \frac{1}{100} \le 1-c \le 1 + \frac{1}{100}$$

$$\frac{1}{1-c} \le \frac{1}{.99}$$

$$|f(x) - P_3(x)| \le \frac{1}{4(99)^4}$$

6

6. Find $\int \frac{dx}{x^2+4x+13} = \int \frac{dx}{(x+2)^2+9} = \frac{1}{3}\tan^{-1}\left(\frac{x+2}{3}\right) + C$

$$\frac{d}{dx}(PA) = \frac{\frac{1}{3} \cdot \frac{1}{3}}{1 + \left(\frac{x+2}{3}\right)^2} = \frac{\frac{1}{9}}{1 + \frac{x^2+4x+4}{9}} = \frac{1}{x^2+4x+13}$$

8