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7. A ball is dropped from the height of 10 feet. Each time it hits the floor it rebounds to $\frac{2}{3}$ its previous height. Find the total distance it travels.

$10\downarrow \ \uparrow 10(\frac{2}{3}) \ \downarrow 10(\frac{2}{3}) \ \uparrow 10(\frac{2}{3})^2 \ \downarrow 10(\frac{2}{3})^2$

Dist $= 10 + 10(\frac{2}{3}) + 10(\frac{2}{3}) + 10(\frac{2}{3})^2 + 10(\frac{2}{3})^2 +$

$= 10 + 20(\frac{2}{3})\left(1 + \frac{2}{3} + (\frac{2}{3})^2 + (\frac{2}{3})^3 + \cdots\right)$

This is a geometric series with ratio $r = \frac{2}{3}$

$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}$ if $-1 < r < 1$

Dist $= 10 + \frac{20(\frac{2}{3})}{1-\frac{2}{3}} = 50$ ft

8. Consider the series $\sum_{k=4}^{\infty} \frac{1}{3^k}$. Give a closed formula for the partial sum $\sum_{k=4}^{n} \frac{1}{3^k}$ Does the series converge? If so, what is the sum of the series?

$s_n = \frac{1}{3^4} + \frac{1}{3^5} + \frac{1}{3^6} + \cdots + \frac{1}{3^n}$

$\frac{1}{3} s_n = \frac{1}{3^5} + \frac{1}{3^6} + \cdots + \frac{1}{3^n} + \frac{1}{3^{n+1}}$

$\frac{2}{3} s_n = \frac{1}{3^4} - \frac{1}{3^{n+1}}$

$s_n = \frac{3}{2}\left(\frac{1}{3^4} - \frac{1}{3^{n+1}}\right)$

$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{3}{2}\left(\frac{1}{3^4} - \frac{1}{3^{n+1}}\right) = \frac{1}{54}$

Yes the series converges to $\frac{1}{54}$.

Or $s_n = \frac{1}{3^4} + \frac{1}{3^4}\cdot\frac{1}{3} + \cdots + \frac{1}{3^4}\frac{1}{3^{n-4}}$

$a + ar + ar^2 + \cdots + ar^m = \frac{a(1-r^{m+1})}{1-r}$

so $s_n = \frac{\frac{1}{3^4}\left(1 - \frac{1}{3^{n-3}}\right)}{1-\frac{1}{3}}$

$a = \frac{1}{3^4}$ $m = n-4$

$r = \frac{1}{3}$