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9. Let $f(x) = \frac{2x+1}{3x-1}$ for $x \neq 1/3$. Find $f^{-1}(x)$.

Let $y = f^{-1}(x)$. $\boxed{\text{Find } y}$

so $f(y) = x$

$\frac{2y+1}{3y-1} = x$

$2y+1 = x(3y-1)$

$y(2-3x) = -x-1$

$y = \frac{-x-1}{2-3x}$

$f^{-1}(x) = \frac{x+1}{3x-2}$ for $x \neq \frac{2}{3}$

ck $f(f^{-1}(x)) = \frac{2\left(\frac{x+1}{3x-2}\right)+1}{3\left(\frac{x+1}{3x-2}\right)-1} = \frac{2x+2+3x-2}{3x+3-3x+2}$

$= \frac{5x}{5} = x$ ✓

$f^{-1}(f(x)) = \frac{\frac{2x+1}{3x-1}+1}{3\left(\frac{2x+1}{3x-1}\right)-2} = \frac{2x+1+3x-1}{6x+3-6x+2}$

$= \frac{5x}{5} = x$ ✓

10. Let $f(x) = x^2 \ln x$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

$f' = x^2 \frac{1}{x} + 2x\ln x = x + 2x\ln x$

$= x(1+2\ln x)$

0 ---- $e^{-\frac{1}{2}}$ +++++

no graph

$f' = 0$ when $1+2\ln x = 0$ $\ln x = -\frac{1}{2}$

$x = e^{-\frac{1}{2}}$

f is dec for $0 < x < e^{-\frac{1}{2}}$
f is inc. for $e^{-\frac{1}{2}} < x$
$\left(e^{-\frac{1}{2}}, -\frac{1}{2e}\right)$ is a loc. min
no loc. max

$f'' = x\left(\frac{2}{x}\right) + 1 + 2\ln x$

$f'' = 3 + 2\ln x$

$f'' = 0$ when $-\frac{3}{2} = \ln x$ so $x = e^{-\frac{3}{2}}$

0 ---- $e^{-\frac{3}{2}}$ +++

no graph

f is cd. for $0 < x < e^{-\frac{3}{2}}$
f is cu. for $e^{-\frac{3}{2}} < x$
$\left(e^{-\frac{3}{2}}, -\frac{3}{2e^3}\right)$ is a pt of i

$\lim_{x \to 0^+} x^2 \ln x = 0$

x^2 wants the answer to be 0
$\ln x$ wants the answer to be $-\infty$
x^2 wins so L'hopital's rule