

PRINT Your Name: _____ Section: _____
 There are 10 problems on 5 pages. Each problem is worth 10 points. each. SHOW your work. **CIRCLE** your answer. NO CALCULATORS! CHECK your answer whenever possible.

$$1. \text{ Find } \int \sin^7 x \, dx = \int (1 - \cos^2 x)^3 \sin x \, dx \stackrel{\substack{\uparrow \\ u = \cos x \\ du = -\sin x \, dx}}{=} -\int (1 - u^2)^3 \, du = -\int (1 - 3u^2 + 3u^4 - u^6) \, du$$

$$= -\left(u - u^3 + \frac{3u^5}{5} - \frac{u^7}{7}\right) + C = -\left(\cos x - \frac{\cos^3 x}{3} + \frac{3}{5}\cos^5 x - \frac{\cos^7 x}{7}\right) + C$$

$$\begin{aligned} \text{check } \frac{d}{dx}(PA) &= -(-\sin x + 3\cos^2 x \sin x - 3\cos^4 x \sin x + \cos^6 x \sin x) \\ &= \sin x (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x) \\ &= \sin x (1 - \cos^2 x)^3 \\ &= \sin^7 x \checkmark \end{aligned}$$

$$2. \text{ Find } \int \sin^4 x \, dx = \frac{1}{4} \int (1 - \cos 2x)^2 \, dx = \frac{1}{4} \int (1 - 2\cos 2x + \cos^2 2x) \, dx$$

$$= \frac{1}{4} \int (1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x)) \, dx = \frac{1}{4} \left(x - \sin 2x + \frac{1}{2}x + \frac{\sin 4x}{8}\right) + C$$

$$\checkmark: \frac{d}{dx}(PA) = \frac{1}{4} \left(-2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2}\right)$$

$$= \frac{1}{4} (1 - 2\cos 2x + \cos^2 2x)$$

$$= \frac{1}{4} (1 - \cos 2x)^2$$

$$= \left(\frac{1 - \cos 2x}{2}\right)^2$$

$$= (\sin^2 x)^2 \checkmark$$