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9. Solve for $x$: $\int_{1/3}^{x} \frac{1}{t}\,dt \quad 2\int_{1}^{x} \frac{1}{t}\,dt$

$\ln t \Big]_{\frac{1}{3}}^{x} = 2 \ln t \Big]_{1}^{x}$

$-\ln \frac{1}{3} = 2\left[\ln x - \underbrace{\ln 1}_{0}\right]$

$\ln 3 = \ln x$

$3 = x$

10. Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the surrounding medium. Thus, if an object is taken from an oven at 300° F and left to cool in a room at 80° F, then its temperature $T$ after $t$ hours will satisfy the differential equation

$$\frac{dT}{dt} = k(T - 80).$$

If the temperature fell to 225° F after one hour, what will it be after 4 hours? (You may leave " ln " in your answer.)

$T(0) = 300$

$T(1) = 225$

Find $T(4)$.

$\int \frac{dT}{T-80} = \int k\,dt$

$\ln|T-80| = kt + C$

$|T-80| = e^{c} e^{kt}$

$T - 80 = \pm e^{c} e^{kt}$

let $K = \pm e^{c}$

$T = 80 + Ke^{kt}$

$300 = T(0) = 80 + Ke^{0}$

$220 = K$

$225 = T(1) = 80 + 220e^{k}$

$145 = 220e^{k}$

$\frac{145}{220} = e^{k}$

$\ln\left(\frac{145}{220}\right) = k$

$T(t) = 80 + 220\, e^{t \ln\left(\frac{145}{220}\right)}$

$T(4) = 80 + 220\, e^{4 \ln\left(\frac{145}{220}\right)}$ °F

If you calc

$T(4) = 80 + 220\left(\frac{145}{220}\right)^{4} \cong 121.5°$ F

I wonder what the cake is made from. It does not seem to cool in a normal manner!