7. Let $f(x) = x \ln x$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

The graph makes sense for $0 < x$ only.

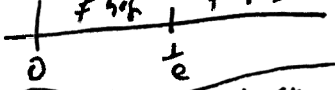
$\lim_{x \to 0^+} x \ln x = 0$ see L'Hopital's rule

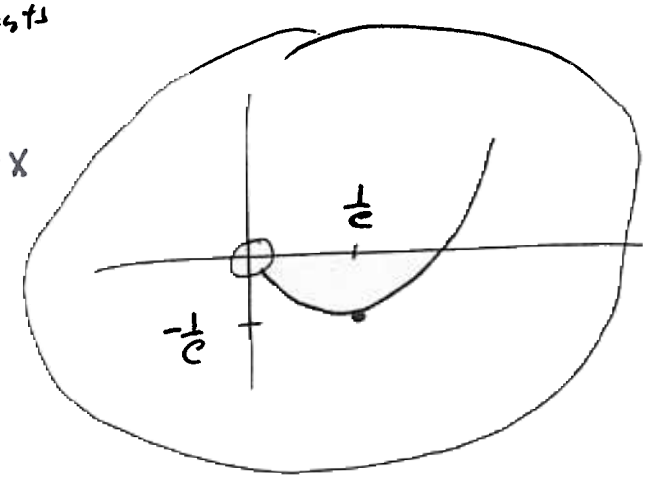
or x is more powerful than $\ln x$

x wants the limit to be 0 $\ln x$ wants the limit to be $-\infty$.

$f' = x \frac{1}{x} + \ln x = 1 + \ln x$

$f' = 0$ when $0 = 1 + \ln x \quad -1 = \ln x \quad e^{-1} = x$





f is inc. for $\frac{1}{e} < x$

f is dec for $0 < x < \frac{1}{e}$

$\left(\frac{1}{e}, -\frac{1}{e}\right)$ is a loc. min

No loc. max

$f'' = \frac{1}{x}$ always positive

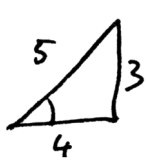
f is c.c.

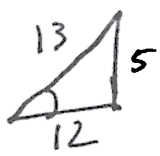
no infl. pts.

~~no loc.~~

8. Simplify $\sin\left[\cos^{-1}\left(\frac{4}{5}\right) - \sin^{-1}\left(\frac{5}{13}\right)\right]$

$\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)\cos\left(\sin^{-1}\left(\frac{5}{13}\right)\right) - \cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right)\sin\left(\sin^{-1}\left(\frac{5}{13}\right)\right)$





$= \frac{3}{5}\frac{12}{13} - \frac{4}{5}\frac{5}{13}$

$= \frac{16}{65}$