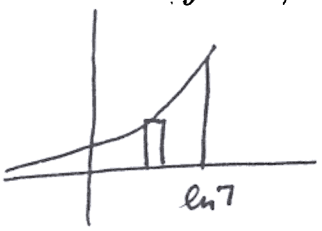
20

4. Find the volume of the solid generated by revolving the region bounded by $y = e^x$, the $x-$ axis, the $y-$ axis, and $x = \ln 7$ about the x-axis.



spin the rectangle get a disc

$\pi r^2 t \quad t = dx \quad r = e^x$

$$\text{Vol} = \pi \int_0^{\ln 7} (e^x)^2 \, dx = \frac{\pi}{2} e^{2x} \Big]_0^{\ln 7} = \frac{\pi}{2}\left(e^{2\ln 7} - e^0\right) = \frac{\pi}{2}(49-1)$$

5. Let $f(x) = 2x^2 - 8x + 9$ for $x \leq 2$ Find $f^{-1}(x)$

Let $y = f^{-1}(x)$

so $f(y) = x$ and $y \leq 2$

$2y^2 - 8y + 9 = x$

$-8y + 9 - x = 0$

$$y = \frac{8 \pm \sqrt{64 - 4(2)(9-x)}}{4}$$

$y = 2 \pm \frac{1}{4}\sqrt{8x-8}$

But $y \leq 2$ so $y = 2 - \frac{1}{2}\sqrt{2x-2}$

so $f^{-1}(x) = 2 - \frac{1}{2}\sqrt{2x-2}$ for $1 \leq x$

6. If $y = x^x$, then find $\frac{dy}{dx}$

$\ln y = x \ln x$

$\frac{1}{y}\frac{dy}{dx} = x\frac{1}{x} + \ln x$

$\frac{dy}{dx} = x^x[1 + \ln x]$