Define the definite integral. Give a complete definition. Be sure to explain all of your notation.

Let $f(x)$ be a continuous function defined on the closed interval $[a, b]$. For each partition P of $[a, b]$ of the form $a = x_0 \leq x_1 \leq \cdots \leq x_n = b$, let M_i be the maximum value of $f(x)$ on the subinterval $[x_{i-1}, x_i]$ and let m_i be the minimum value of $f(x)$ on $[x_{i-1}, x_i]$. If there is exactly one number with

$$\sum_{i=1}^{n} m_i(x_i - x_{i-1}) \leq \text{this number} \leq \sum_{i=1}^{n} M_i(x_i - x_{i-1}),$$

as P varies over all partitions of $[a, b]$, then this number is called the definite integral of f on $[a, b]$ and this number is denoted $\int_a^b f(x)dx$.