

PRINT Your Name: \_\_\_\_\_

**Quiz 7 — February 28, 2014 – Section 8 – 10:50 – 11:40**

**Remove everything from your desk except this page and a pencil or pen.**

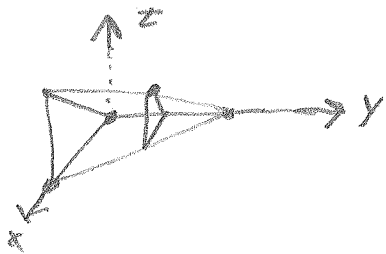
The solution will be posted soon after the quiz is given.

**Circle** your answer. **Show your work.** Your work must be correct and **coherent**.

The quiz is worth 5 points.

Consider the solid  $S$ . The base of  $S$  is the triangular region in the  $xy$ -plane with vertices  $(0, 0)$ ,  $(1, 0)$ , and  $(0, 1)$ . The cross sections of  $S$  which are perpendicular to the  $y$ -axis are equilateral triangles. Find the volume of  $S$ . You must draw a meaningful picture.

Section 8

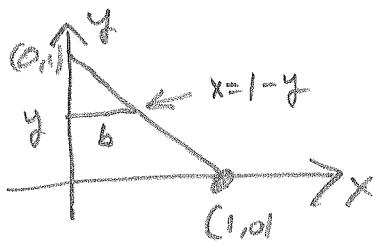


Chop the  $y$ -axis from 0 to  $y$ . For each  $y$ -coordinate  $y$  consider the slice of the solid whose  $y$ -coordinate is  $y$ . This slice looks like an equilateral triangle with thickness



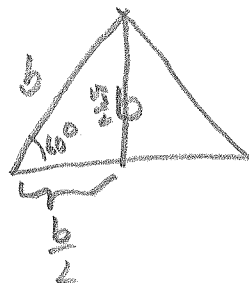
The volume of the slice is the area of the face times the thickness  $= \frac{1}{2} b h t$

where  $b$  is the base  $h$  is the height and  $t$  is the thickness, well  $t = dy$ . We can figure out  $b$  in terms of  $y$



$$b = 1 - y$$

Use Trig to find  $h$



$$h = \frac{\sqrt{3}}{2} b = \frac{\sqrt{3}}{2} (1-y)$$

The vol of one slice is  $\frac{1}{2} b h t = \frac{\sqrt{3}}{4} (1-y)^2 dy$

$$\text{The vol of the solid is } \int_0^1 \frac{\sqrt{3}}{4} (1-y)^2 dy = -\frac{\sqrt{3}}{4} \left[ \frac{(1-y)^3}{3} \right]_0^1 = \boxed{\frac{1}{4\sqrt{3}}}$$

Check: The solid is a pyramid with base of area  $\frac{\sqrt{3}}{4}$ . so the volume is  $\frac{1}{3} \left( \frac{\sqrt{3}}{4} \right) (ht) = \frac{1}{3} \frac{\sqrt{3}}{4}$