April 13, 2011 Quizzes 12 and 13 Sect 4 9:05

1 Does the series $\sum_{k=2}^{\infty} k^2 e^{-k}$ converge? **Justify your answer very thoroughly.**

Use the ratio test. We compute

$$\rho = \lim_{k\to\infty} \frac{(k+1)^2 e^{-(k+1)}}{k^2 e^{-k}} = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^2 e^{-(k+1)} e^{k} = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^2 e^{-1} = \frac{1}{e}.$$

Thus, $\rho < 1$. The ratio test guarantees that $\sum_{k=2}^{\infty} k^2 e^{-k}$ converges.

2 Evaluate the indefinite integral $\int \frac{t}{1-t^8}\,dt$ as a power series. What is the radius of convergence?

Answer. The geometric series $\sum_{n=0}^{\infty} (t^8)^n$ converges to $\frac{1}{1-t^8}$ for $-1 < t^8 < 1$. Notice that $-1 < t^8 < 1$ if and only if $-1 < t < 1$. So

$$\sum_{n=0}^{\infty} t^{8n} = \frac{1}{1-t^8} \quad \text{for } -1 < t < 1.$$

Multiply by t to see that

$$\sum_{n=0}^{\infty} t^{8n+1} = \frac{t}{1-t^8} \quad \text{for } -1 < t < 1.$$

Integrate to see that

$$\boxed{\sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2} + C = \int \frac{t}{1-t^8}\,dt \quad \text{for } -1 < t < 1.}$$