April 13, 2011 Quizzes 12 and 13 Section 3 8:00

1. Does the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

We apply the integral test. We see that $f(x) = \frac{1}{x\sqrt{\ln x}}$ is a positive decreasing function. The integral test guarantees that the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ and the integral $\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}dx$ both converge or both diverge. We compute the integral. Let $u = \ln x$. It follows that $du = \frac{1}{x}dx$. When $x = 2$, we have $u = \ln 2$. When $x$ goes to infinity, then $u$ also goes to infinity. We have

$$\int_2^{\infty} \frac{1}{x\sqrt{\ln x}}dx = \int_{\ln 2}^{\infty} u^{-1/2}du = \lim_{b\to\infty} 2\sqrt{u}\Big|_{\ln 2}^{b} = \lim_{b\to\infty} (2\sqrt{b} - 2\sqrt{\ln 2}) = \infty.$$

The integral diverges. Thus, the series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ also diverges.

2. Evaluate the indefinite integral $\int \frac{t}{1-t^8}dt$ as a power series. What is the radius of convergence?

**Answer.** The geometric series $\sum_{n=0}^{\infty}(t^8)^n$ converges to $\frac{1}{1-t^8}$ for $-1 < t^8 < 1$. Notice that $-1 < t^8 < 1$ if and only if $-1 < t < 1$. So

$$\sum_{n=0}^{\infty} t^{8n} = \frac{1}{1-t^8} \quad \text{for } -1 < t < 1.$$

Multiply by $t$ to see that

$$\sum_{n=0}^{\infty} t^{8n+1} = \frac{t}{1-t^8} \quad \text{for } -1 < t < 1.$$

Integrate to see that

$$\boxed{\sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2} + C = \int \frac{t}{1-t^8}dt \quad \text{for } -1 < t < 1.}$$