

Quiz 9, February 9, 2016

Find $\int \sec^4 \theta d\theta$.

Answer: We have to integrate an even power of $\sec \theta$ times some power of $\tan \theta$. We save $\sec^2 \theta$, convert all remaining $\sec \theta$'s into $\tan \theta$'s (using $\sec^2 \theta = \tan^2 \theta + 1$) and let $u = \tan \theta$ (and so $du = \sec^2 \theta d\theta$). So

$$\int \sec^4 \theta d\theta = \int (\tan^2 \theta + 1) \sec^2 \theta d\theta = \int (u^2 + 1) du = \frac{u^3}{3} + u + C = \boxed{\frac{\tan^3 \theta}{3} + \tan \theta + C}$$

Check: The derivative of the proposed answer is

$$\tan^2 \theta \sec^2 \theta + \sec^2 \theta = \sec^2 \theta (\tan^2 \theta + 1) = \sec^4 \theta. \checkmark$$