Quiz 8, February 2, 2015

Find $\int \frac{dx}{(x^2-1)^{3/2}}$. (In this problem $1 < x$.)

Answer: Let $x = \sec\theta$. It follows that $dx = \sec\theta\tan\theta d\theta$ and $x^2 - 1 = \sec^2\theta - 1 = \tan^2\theta$. The original integral is

$$\int \frac{\sec\theta\tan\theta d\theta}{\tan^3\theta} = \int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \csc\theta\cot\theta d\theta = -\csc\theta + C.$$

Draw a right triangle with $x = \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$. So this triangle has x on the hypotenuse, 1 on the adjacent, and $\sqrt{x^2-1}$ on the opposite. So $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{x}{\sqrt{x^2-1}}$. Our integral is equal to

$$-\csc\theta + C = \boxed{\frac{-x}{\sqrt{x^2-1}} + C}.$$

Check: The derivative of the proposed answer is

$-x(-1/2)(x^2-1)^{-3/2}(2x) - (x^2-1)^{-1/2} = (x^2-1)^{-3/2}(x^2 - (x^2-1)) = (x^2-1)^{-3/2}$. ✓