

### Quiz 8, September 15, 2016

Find  $\int \frac{dx}{(x^2-1)^{3/2}}$ . (In this problem  $1 < x$ .)

**Answer:** Let  $x = \sec \theta$ . It follows that  $dx = \sec \theta \tan \theta d\theta$  and  $x^2 - 1 = \sec^2 \theta - 1 = \tan^2 \theta$ . The original integral is

$$\int \frac{\sec \theta \tan \theta d\theta}{\tan^3 \theta} = \int \frac{\cos \theta}{\sin^2 \theta} d\theta = \int \csc \theta \cot \theta d\theta = -\csc \theta + C.$$

Draw a right triangle with  $x = \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$ . So this triangle has  $x$  on the hypotenuse, 1 on the adjacent, and  $\sqrt{x^2 - 1}$  on the opposite. So  $\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{x}{\sqrt{x^2 - 1}}$ . Our integral is equal to

$$-\csc \theta + C = \boxed{\frac{-x}{\sqrt{x^2 - 1}} + C}.$$

**Check:** The derivative of the proposed answer is

$$-x(-1/2)(x^2 - 1)^{-3/2}(2x) - (x^2 - 1)^{-1/2} = (x^2 - 1)^{-3/2}(x^2 - (x^2 - 1)) = (x^2 - 1)^{-3/2}. \checkmark$$