

Quiz 6, January 27, 2015

Find $\int e^{2x} \cos 3x dx$.

Answer: Try integration by parts. Let $u = e^{2x}$ and $dv = \cos 3x dx$. We compute $du = 2e^{2x} dx$ and $v = \frac{1}{3} \sin 3x$. So

$$\int e^{2x} \cos 3x dx = \int u dv = uv - \int v du = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x dx.$$

We try integration by parts again with $u = e^{2x}$ and $dv = \sin 3x dx$. We compute $du = 2e^{2x} dx$ and $v = -\frac{1}{3} \cos 3x$. We see that

$$\int e^{2x} \cos 3x dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \left[-\frac{1}{3} e^{2x} \cos 3x + \frac{2}{3} \int e^{2x} \cos 3x dx \right].$$

Add $\frac{4}{9} \int e^{2x} \cos 3x dx$ to both sides to obtain

$$\frac{13}{9} \int e^{2x} \cos 3x dx = \frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \frac{(-1)}{3} e^{2x} \cos 3x.$$

Multiply both sides by $\frac{9}{13}$ to see

$$\begin{aligned} \int e^{2x} \cos 3x dx &= \frac{9}{13} \left[\frac{1}{3} e^{2x} \sin 3x - \frac{2}{3} \frac{(-1)}{3} e^{2x} \cos 3x \right] \\ &= \boxed{\frac{e^{2x}}{13} [3 \sin 3x + 2 \cos 3x] + C} \end{aligned}$$

Check: The derivative of the proposed answer is

$$\frac{e^{2x}}{13} [(9 \cos 3x - 6 \sin 3x) + (6 \sin 3x + 4 \cos 3x)] = e^{2x} \cos 3x \checkmark$$