

**Quiz 28, April 5, 2016**

Find the Taylor polynomials of orders 0, 1, 2, and 3 for  $f(x) = \sqrt{x}$  about  $a = 4$ .

**Answer:** We see that

$$\begin{aligned} f(x) &= x^{1/2} & f(4) &= 2 \\ f'(x) &= \frac{1}{2}x^{-1/2} & f'(4) &= \frac{1}{4} \\ f''(x) &= -\frac{1}{4}x^{-3/2} & f''(4) &= -\frac{1}{32} \\ f'''(x) &= \frac{3}{8}x^{-5/2} & f'''(4) &= \frac{3}{256} \end{aligned}$$

The Taylor polynomial of order  $n$  for  $f(x)$  about  $a$  is

$$P_n(x) = f(a) + f'(a)(x-a) + f''(a)\frac{(x-a)^2}{2!} + \cdots + f^{(n)}(a)\frac{(x-a)^n}{n!};$$

consequently in our problem

$$P_0(x) = 2$$

$$P_1(x) = 2 + \frac{1}{4}(x-4)$$

$$P_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32}\frac{(x-4)^2}{2!}$$

$$P_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32}\frac{(x-4)^2}{2!} + \frac{3}{256}\frac{(x-4)^3}{3!}$$