Quiz 20, March 16, 2016

Consider the sequence $a_1 = 2$ and $a_{n+1} = \frac{72}{1+a_n}$. Asuume the the sequence converges. Find the limit of the sequence.

Answer: We are told that $\lim_{n\to\infty} a_n$ exists; let $L = \lim_{n\to\infty} a_n$. Take $\lim_{n\to\infty}$ of both sides of

$$a_{n+1} = \frac{72}{1+a_n}$$

to obtain

$$L = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{72}{1+a_n} = \frac{72}{1+L}.$$

Solve for L: $L(1+L) = 72$ or $L^2 + L - 72 = 0$ or $(L-8)(L+9) = 0$; so L is 8 or -9. On the other hand, every a_n is positive because a_1 is positive and to get a new a from and old a one adds 1 and divides into 72. Both of these processes gives a positive answer if the input is positive. So, L must be $\boxed{8}$.