Quiz 19, November 3, 2016

Does the series $\sum\limits_{k=1}^{\infty} \frac{\ln k}{k^3}$ converge? Justify your answer.

Answer: Observe that $\ln(k) < k$ for $1 \le k$. It follows that $\frac{\ln k}{k^3} < \frac{k}{k^3}$ for $1 \le k$. Of course, $\frac{k}{k^3} = \frac{1}{k^2}$. The series $\sum\limits_{k=1}^{\infty} \frac{1}{k^2}$ is the p-series with $p = 2$. This p-series converges because $1 < 2$.

Thus, the original series, $\sum\limits_{k=1}^{\infty} \frac{\ln k}{k^3}$, converges by a straight application of the comparison test.