Quiz 17, March 2, 2016

Consider the series $\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$.

(a) Find a closed formula for the partial sum $s_N = \sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$.

(b) Does the series $\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$ converge? If so, find its sum.

Answer:

(a) We see that

$$\begin{aligned} s_N &= \sum_{n=1}^{\infty} \ln(\tfrac{n}{n+1}) \\ &= \ln(\tfrac{1}{2}) + \ln(\tfrac{2}{3}) + \ln(\tfrac{3}{4}) + \cdots + \ln(\tfrac{N}{N+1}) \\ &= [\ln(1) - \ln(2)] + [\ln(2) - \ln(3)] + [\ln(3) - \ln(4)] + \tfrac{2}{3}) + \ldots \\ &\quad \cdots + [\ln(N-1) - \ln(N)] + [\ln(N) - \ln(N+1)] \\ &= \ln(1) - \ln(N+1) = \boxed{-\ln(N+1)}. \end{aligned}$$

(b) The sum of the series is the limit of the sequence of partial sums and this is

$$\lim_{N\to\infty} -\ln(N+1) = -\infty.$$

We conclude that

$$\boxed{\text{the series } \sum_{n=1}^{\infty} \ln(\tfrac{n}{n+1}) \text{ diverges to } -\infty.}$$