Quiz 16, October 20, 2016

Does the series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ converge? Justify your answer.

Answer: We see that the N^{th} partial sum of the series is

$$s_N = \sum_{n=1}^{N} \ln\left(\frac{n}{n+1}\right) = (\ln(1) - \ln(2)^{\dagger}) + (\ln(2)^{\dagger} - \ln(3)^{\dagger\dagger}) + (\ln(3)^{\dagger\dagger} - \ln(4)^{\dagger\dagger\dagger}) + {}^{\dagger\dagger\dagger}\ldots$$

$$\ldots^{\ddagger\ddagger\ddagger} + (\ln(N-2)^{\ddagger\ddagger\ddagger} - \ln(N-1)^{\ddagger\ddagger}) + (\ln(N-1)^{\ddagger\ddagger} - \ln(N)^{\ddagger}) + (\ln(N)^{\ddagger} - \ln(N+1))$$

$$= \ln(1) - \ln(N+1) = -\ln(N+1).$$

The sum of the series is the limit of the sequence of partial sums

$$= \lim_{N\to\infty} s_N = \lim_{N\to\infty} -\ln(N+1) = -\infty.$$

Thus the series $\sum_{k=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ diverges.