

### Quiz 10, September 21, 2016

Find  $\int \frac{dx}{1-x^2}$ .

**Answer:** Observe that  $\frac{1}{1-x^2} = \frac{-1}{x^2-1} = \frac{-1}{(x-1)(x+1)}$ . We use the technique of partial fractions and look for numbers  $A$  and  $B$  with

$$\frac{-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}.$$

Multiply both sides by  $(x-1)(x+1)$  to obtain

$$-1 = A(x+1) + B(x-1)$$

Plug in  $x = 1$  to learn that  $A = \frac{-1}{2}$ . Plug in  $x = -1$  to learn that  $B = \frac{1}{2}$ . We have calculated that

$$\frac{1}{1-x^2} = \frac{-\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}.$$

Before going any further, let's make sure this is correct. We see that the right side is

$$\frac{1}{2} \left[ \frac{-(x+1) + (x-1)}{(x-1)(x+1)} \right] = \frac{-1}{x^2-1} = \frac{1}{1-x^2},$$

as desired. We now compute

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \left[ \int \frac{-1}{x-1} + \frac{1}{x+1} \right] = \boxed{\frac{1}{2} [-\ln|x-1| + \ln|x+1|] + C}$$