Math 142, Exam 1, Fall 2015, Solutions

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. **Find** $\int \sin^2 x \cos^3 x dx$ **. Please check your answer.**

Save $\cos x$. Convert the rest of the $\cos x$'s to $\sin x$. The integral equals

$$\int \sin^2 x(1 - \sin^2 x) \cos x dx.$$

Let $u = \sin x$. It follows that $du = \cos x dx$. Thus,

$$\int \sin^2 x \cos^3 x dx = \int u^2(1 - u^2)du = \int (u^2 - u^4)du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C = \boxed{\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C}.$$

Check. The derivative of the proposed answer is

$$\sin^2 x \cos x - \sin^4 x \cos x = \sin^2 x \cos x(1 - \sin^2 x) = \sin^2 x \cos x \cos^2 x$$

$$= \sin^2 x \cos^3 x. \checkmark$$

2. **Find** $\int \frac{dx}{\sqrt{x^2 + 2x + 17}}$ **. Please check your answer.**

$$\int \frac{dx}{\sqrt{x^2 + 2x + 17}} = \int \frac{dx}{\sqrt{x^2 + 2x + \square + 17 - \square}}.$$

Fill in the box with $(\text{one half of } 2)^2$ because $x^2 + 2x + 1 = (x + 1)^2$

$$\int \frac{dx}{\sqrt{x^2 + 2x + 17}} = \int \frac{dx}{\sqrt{x^2 + 2x + \boxed{1} + 17 - \boxed{1}}} = \int \frac{dx}{\sqrt{(x + 1)^2 + 16}}.$$

Let $x+1=4\tan\theta$. It follows that

$$(x+1)^2+16=16\tan^2\theta+16=16(\tan^2\theta+1)=16\sec^2\theta$$

and $dx=4\sec^2\theta d\theta$. So,

$$\int\frac{dx}{\sqrt{x^2+2x+17}}=\int\frac{4\sec^2\theta}{4\sec\theta}d\theta=\int\sec\theta d\theta=\ln|\sec\theta+\tan\theta|+C$$

$$=\ln\left|\frac{\sqrt{x^2+2x+17}}{4}+\frac{x+1}{4}\right|+C=\ln|\sqrt{x^2+2x+17}+x+1|-\ln 4+C$$

$$=\boxed{\ln|\sqrt{x^2+2x+17}+x+1|+K},$$

where $K=-\ln 4+C$.

Check. The derivative of the proposed answer is

$$\frac{\frac{2x+2}{2\sqrt{x^2+2x+17}}+1}{\sqrt{x^2+2x+17}+x+1}=\frac{\frac{2(x+1)}{2\sqrt{x^2+2x+17}}+1}{\sqrt{x^2+2x+17}+x+1}=\frac{\frac{(x+1)}{\sqrt{x^2+2x+17}}+1}{\sqrt{x^2+2x+17}+x+1}$$

$$=\frac{(x+1)+\sqrt{x^2+2x+17}}{\sqrt{x^2+2x+17}(\sqrt{x^2+2x+17}+x+1)}=\frac{1}{\sqrt{x^2+2x+17}}.\ \checkmark$$

3. **Find** $\int\frac{xdx}{e^{3x}}$ **. Please check your answer.**

Use integration by parts. Let $u=x$ and $dv=e^{-3x}dx$. Compute $du=dx$ and $v=\frac{-1}{3}e^{-3x}$. It follows that

$$\int\frac{xdx}{e^{3x}}=uv-\int vdu=\frac{-x}{3}e^{-3x}+\frac{1}{3}\int e^{-3x}dx=\boxed{\frac{-x}{3}e^{-3x}-\frac{1}{9}e^{-3x}+C}$$

Check. The derivative of the proposed answer is

$$(-3)\frac{-x}{3}e^{-3x}+\frac{-1}{3}e^{-3x}+\frac{1}{3}e^{-3x}=xe^{-3x}.\ \checkmark$$

4. **Find** $\int e^{7x}\sin x dx$ **. Please check your answer.**

Use parts. Let $u = e^{7x}$ and $dv = \sin x dx$. It follows that $du = 7e^{7x}$ and $v = -\cos x$. Thus,

$$\int e^{7x}\sin x dx = -e^{7x}\cos x + 7\int e^{7x}\cos x dx.$$

Use parts again. This time let $u = e^{7x}$ and $dv = \cos x dx$. We compute $du = 7e^{7x}dx$ and $v = \sin x$. We now have

$$\int e^{7x}\sin x dx = -e^{7x}\cos x + 7\left(e^{7x}\sin x - 7\int e^{7x}\sin x dx\right).$$

Add $49\int e^{7x}\sin x dx$ to both sides to learn that

$$50\int e^{7x}\sin x dx = -e^{7x}\cos x + 7e^{7x}\sin x + C.$$

$$\int e^{7x}\sin x dx = \frac{1}{50}\left(-e^{7x}\cos x + 7e^{7x}\sin x\right) + K,$$

where $K = C/50$.

$$\boxed{\int e^{7x}\sin x dx = \frac{e^{7x}}{50}\left(-\cos x + 7\sin x\right) + K.}$$

Check. The derivative of the proposed answer is

$$\frac{e^{7x}}{50}(\sin x + 7\cos x) + 7\frac{e^{7x}}{50}\left(-\cos x + 7\sin x\right) = 50\frac{e^{7x}}{50}\sin x = e^{7x}\sin x. \checkmark$$

5. **Find** $\int \frac{x^4}{\sqrt[3]{1+x^5}}dx$ **. Please check your answer.**

Let $u = 1 + x^5$. It follows that $du = 5x^4dx$ and

$$\int \frac{x^4}{\sqrt[3]{1+x^5}}dx = \frac{1}{5}\int u^{-1/3}du = \frac{1}{5}\frac{3}{2}u^{2/3} + C = \boxed{\frac{3}{10}(1+x^5)^{2/3} + C}.$$

Check. The derivative of the proposed answer is

$$\frac{3}{10}\frac{2}{3}(1+x^5)^{-1/3}5x^4 = x^4(1+x^5)^{-1/3}. \checkmark$$