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Find $\lim\limits_{x\to 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2}$. Explain carefully which facts you are using.

ANSWER: We see that

$$\lim_{x\to 0} \frac{\tan 3x^2 + \sin^2 5x}{x^2} = \lim_{x\to 0} \left(\frac{3}{\cos 3x^2} \frac{\sin 3x^2}{3x^2} + 25 \frac{\sin 5x}{5x} \frac{\sin 5x}{5x} \right).$$

We know that $\lim\limits_{t\to 0} \frac{\sin t}{t} = 1$. We apply this fact with t replaced by $3x^2$ and later with t replaced by $5x$. We know that the limit of a sum is the sum of the limits and the limit of a product is the product of the limits. The answer to our problem is

$$3 \cdot 1 + 25 \cdot 1 \cdot 1 = \boxed{28}.$$