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Quiz for November 10, 2005

Let $f(x) = \frac{x^3-4x-8}{x+2}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Find all interesting asymptotes. Graph $y = f(x)$.

ANSWER: We divide to see that $f(x) = x^2 - 2x - \frac{8}{x+2}$. It is now apparent that $\lim\limits_{x\to -2^+} f(x) = -\infty$ and $\lim\limits_{x\to -2^-} f(x) = +\infty$. We conclude that

$$\boxed{x = -2 \text{ is a vertical asymptote of } y = f(x).}$$

Also

$$\lim_{x\to\infty} |f(x) - (x^2 - 2x)| = \lim_{x\to\infty} |-\frac{8}{x+2}| = 0$$

and

$$\lim_{x\to-\infty} |f(x) - (x^2 - 2x)| = \lim_{x\to-\infty} |-\frac{8}{x+2}| = 0;$$

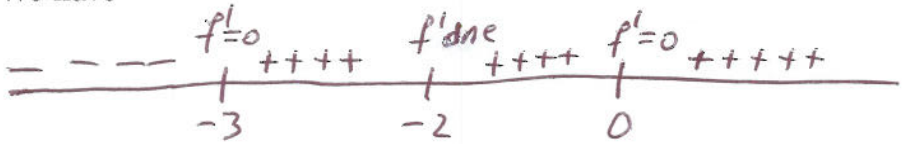
so,

$$\boxed{\text{the graph of } y = f(x) \text{ is asymptotic to the graph of } y = x^2 - 2x.}$$

We calculate

$$f'(x) = 2x - 2 + \frac{8}{(x+2)^2} = \frac{2x^3 + 6x^2}{(x+2)^2} = \frac{2x^2(x+3)}{(x+2)^2}.$$

So, $f'(x)$ is zero when $x = 0$ or $x = -3$ and $f'(x)$ does not exist when $x = -2$. We have



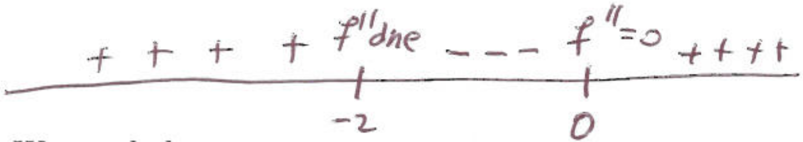
We conclude

$$\boxed{\begin{array}{l} f(x) \text{ is increasing for } -3 < x, \\ f(x) \text{ is decreasing for } x < -3, \\ (-3, 23) \text{ is the only local minimum point} \\ \text{there are no local maximum points} \end{array}}$$

We also compute

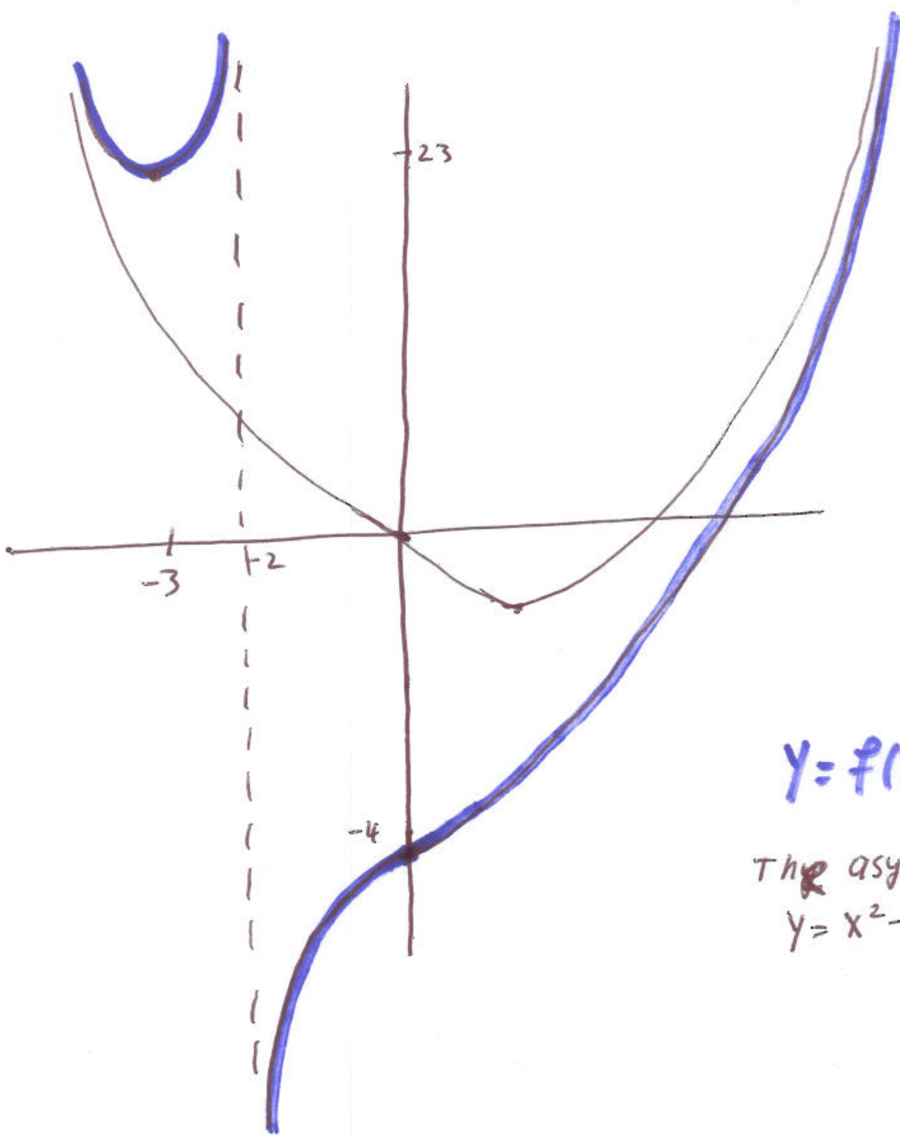
$$f''(x) = 2 + \frac{-16}{(x+2)^3} = \frac{2(x^3 + 6x^2 + 12x + 8) - 16}{(x+2)^3} = \frac{2x(x^2 + 6x + 12)}{(x+2)^3}.$$

Be sure to notice that the factor $x^2 + 6x + 12$ is never zero. Thus, this factor is always positive. We see that $f''(x)$ is zero only at $x = 0$, and $f''(x)$ does not exist at $x = -2$. We have



We conclude

$$\boxed{\begin{array}{l} f(x) \text{ is concave up for } x < -2, \text{ also for } 0 < x, \\ f(x) \text{ is concave down for } -2 < x < 0, \\ (0, -4) \text{ is the only point of inflection} \end{array}}$$



y = f(x) is blue

The asymptot $y = x^2 - 2x$ is in black.