PRINT your name ______________________________

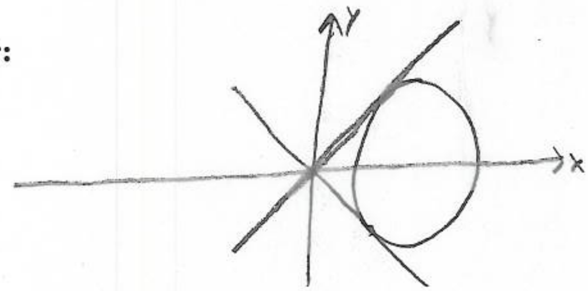
Quiz for March 3, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.
Circle your answer. Show your work.
The quiz is worth 5 points.

Find equations for two lines through the origin that are tangent to

(*) $$2x^2 - 4x + y^2 + 1 = 0.$$

Answer:



Let (a, b) be a point on the graph of

$$2x^2 - 4x + y^2 + 1 = 0$$

with the property that the tangent line passes through the origin. We automatically know that $2a^2 - 4a + b^2 + 1 = 0$. We also know that the slope of the line joining (a, b) to $(0, 0)$ is $\frac{dy}{dx}|_{(a,b)}$. The line joining (a, b) to $(0, 0)$ has slope $\frac{b}{a}$. We take $\frac{d}{dx}$ of both sides of (*) to see that

$$4x - 4 + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = 4 - 4x$$

$$\frac{dy}{dx} = \frac{4 - 4x}{2y} = \frac{2 - 2x}{y}.$$

We solve the equations

$$2a^2 - 4a + b^2 + 1 = 0 \quad \text{and} \quad \frac{b}{a} = \frac{2 - 2a}{b}$$

simultaneously. We solve the equations

$$2a^2 - 4a + b^2 + 1 = 0 \quad \text{and} \quad b^2 = 2a - 2a^2$$

simultaneously. We solve the equations

$$2a^2 - 4a + (2a - 2a^2) + 1 = 0 \quad \text{and} \quad b^2 = 2a - 2a^2$$

simultaneously. We solve the equations

$$-2a + 1 = 0 \quad \text{and} \quad b^2 = 2a - 2a^2$$

simultaneously. We solve the equations

$$a = \frac{1}{2} \quad \text{and} \quad b^2 = 2a - 2a^2$$

simultaneously. So, $a = \frac{1}{2}$ and $b^2 = \frac{1}{2}$. The two points are $(\frac{1}{2}, \frac{1}{\sqrt{2}})$ and $(\frac{1}{2}, -\frac{1}{\sqrt{2}})$. We see that $\frac{dy}{dx}|_{(\frac{1}{2},\frac{1}{\sqrt{2}})} = \sqrt{2}$ and $\frac{dy}{dx}|_{(\frac{1}{2},-\frac{1}{\sqrt{2}})} = -\sqrt{2}$. The tangent lines are $y - \frac{1}{\sqrt{2}} = \sqrt{2}(x - \frac{1}{2})$ and $y + \frac{1}{\sqrt{2}} = -\sqrt{2}(x - \frac{1}{2})$; in other words,

$$\boxed{y = \sqrt{2}x \quad \text{and} \quad y = -\sqrt{2}x}.$$