PRINT your name _____

Quiz for February 10, 2009 – 9:30 section

Remove everything from your desk except this page and a pencil or pen.
Circle your answer. Show your work.
The quiz is worth 5 points.

Find a nonzero value for the constant k that makes

$$f(x) = \begin{cases} \frac{\tan kx}{x} & \text{if } x < 0 \\ 3x + 2k^2 & \text{if } 0 \leq x \end{cases}$$

be continuous at $x = 0$.

Answer: We see that

$$\lim_{x \to 0^+} f(x) = 2k^2 \quad \text{and} \quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin kx}{kx} \cdot \frac{k}{\cos x} = k$$

In order for $f(x)$ to be continuous at $x = 0$, we need

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x).$$

In other words, we must have

$$2k^2 = k.$$

So,

$$k(2k - 1) = 0$$

and $k = 0$ or $\boxed{k = 1/2}$. Be sure to notice that our choice of k yields $f(0) = \lim_{x \to 2} f(0) = 1/2$.