Recitation Time _____ PRINT your name _____

## Math 141,  Exam 1,  Solutiuons   Spring 2009

The exam is worth a total of 50 points.  There are 7 questions on 3 pages. SHOW your work.  Make your work be coherent and clear.  Write in complete sentences whenever this is possible.  $\boxed{CIRCLE}$ your answer.  **CHECK** your answer whenever possible. **No Calculators.**

I will post the solutions on my website a few hours after the exam is finished.

1. (7 points) **Parameterize the curve pictured below.  Use $t$ as your parameter with $0 \le t \le 2$.  The point that corresponds to $t = 0$ is $(0,0)$.  The point that corresponds to $t = 1$ is $(1,1)$.  The point that corresponds to $t = 2$ is $(0,1)$.  (Note: Each part of the curve that looks like a line segment is a line segment.)**

The first leg of the trip is on the line $y = x$.  If we take $x = t$ and $y = t$, then we walk off this line segment from $(0,0)$ to $(1,1)$ as $t$ goes from $0$ to $1$.  The second leg of this trip is on the line $y = 1$.  At $t = 1$, we have $x = 1$.  At $t = 2$, we have $x = 0$.  The function $x(t) = 2 - t$ has the desired effect.  The curve is parameterized by

$$\boxed{x(t) = \begin{cases} t & \text{if } 0 \le t \le 1 \\ 2 - t & \text{if } 1 \le t \le 2 \end{cases} \qquad y(t) = \begin{cases} t & \text{if } 0 \le t \le 1 \\ 1 & \text{if } 1 \le t \le 2. \end{cases}}$$
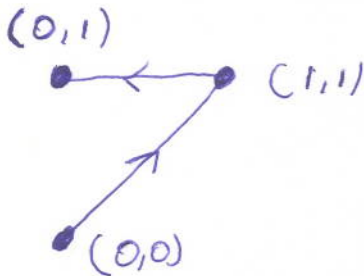


2. (7 points) **Express $\sin(\theta + \varphi)$ in terms of $\sin\theta$, $\sin\varphi$, $\cos\theta$, and $\cos\varphi$.**

One of the five trig facts is:

$$\boxed{\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi.}$$

3. (7 points) **Compute** $\lim\limits_{x \to 6+} \frac{x+6}{x^2-36}$ .

Factor the denominator to see that

$$\lim_{x \to 6+} \frac{x+6}{x^2-36} = \lim_{x \to 6+} \frac{x+6}{(x+6)(x-6)} = \lim_{x \to 6+} \frac{1}{(x-6)} = \boxed{+\infty.}$$

4. (7 points) **Compute** $\lim\limits_{x \to 6-} \frac{36-x}{6-\sqrt{x}}$ .
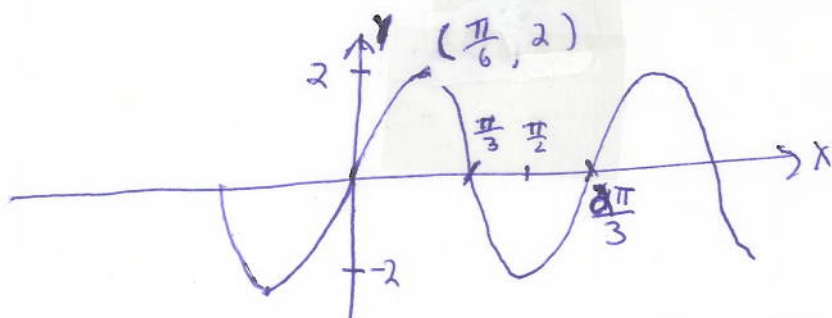
Multiply top and bottom by $6+\sqrt{x}$ to see that

$$\lim_{x \to 6-} \frac{36-x}{6-\sqrt{x}} = \lim_{x \to 6-} \frac{(36-x)(6+\sqrt{x})}{(6-\sqrt{x})(6+\sqrt{x})} = \lim_{x \to 6-} \frac{(36-x)(6+\sqrt{x})}{36-x}$$

$$= \lim_{x \to 6-} (6+\sqrt{x}) = \boxed{6+\sqrt{6}.}$$

5. (7 points) **Let** $f(x) = 2x^2 + 3$. **Find** $\frac{f(a)-f(b)}{a-b}$ **and simplify as much as possible.**

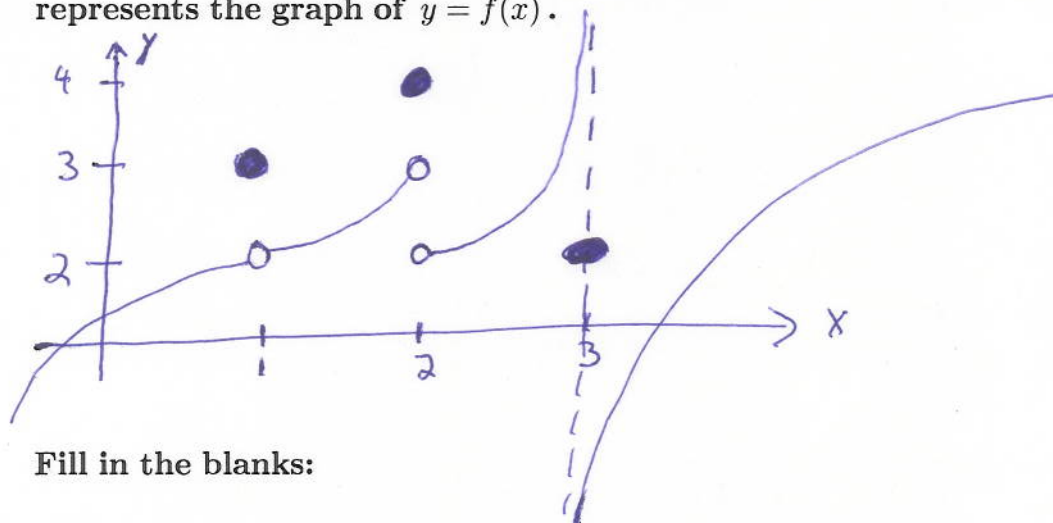We have

$$\frac{f(a)-f(b)}{a-b} = \frac{(2a^2+3)-(2b^2+3)}{a-b} = \frac{2a^2-2b^2}{a-b} = \frac{2(a+b)(a-b)}{a-b} = \boxed{2(a+b).}$$

6. (7 points) **Graph** $y = 2\sin(3x)$. **Identify a few points on the graph.**



7. (8 points) (The penalty for each mistake is four points.) The picture represents the graph of $y = f(x)$.



**Fill in the blanks:**

$f(1) = 3$     $\lim\limits_{x \to 1^+} f(x) = 2$     $\lim\limits_{x \to 1^-} f(x) = 2$     $\lim\limits_{x \to 1} f(x) = 2$

$f(2) = 4$     $\lim\limits_{x \to 2^+} f(x) = 2$     $\lim\limits_{x \to 2^-} f(x) = 3$     $\lim\limits_{x \to 2} f(x) = $ Does not exist

$f(3) = 2$     $\lim\limits_{x \to 3^+} f(x) = -\infty$     $\lim\limits_{x \to 3^-} f(x) = +\infty$     $\lim\limits_{x \to 3} f(x) = $ Does not exist