Math 141, 1995, Final Exam

PRINT Your Name:________________________________ There are 19 problems on 10 pages. The exam is worth 200 points. Problems 1 and 3 are each worth 15 points. Each of the other problems is worth 10 points. SHOW your work. *CIRCLE* your answer. **NO CALCULATORS!!!**

1. (The penalty for each mistake is five points.) The picture represents the graph of $y = f(x)$.
   (a) Fill in the blanks:

$$f(1) = \_\_ \quad \lim_{x\to 1^+} f(x) = \_\_ \quad \lim_{x\to 1^-} f(x) = \_\_ \quad \lim_{x\to 1} f(x) = \_\_$$
$$f(2) = \_\_ \quad \lim_{x\to 2^+} f(x) = \_\_ \quad \lim_{x\to 2^-} f(x) = \_\_ \quad \lim_{x\to 2} f(x) = \_\_$$
$$f(3) = \_\_ \quad \lim_{x\to 3^+} f(x) = \_\_ \quad \lim_{x\to 3^-} f(x) = \_\_ \quad \lim_{x\to 3} f(x) = \_\_$$

   (b) Where is f discontinuous?
   (c) Where is f not differentiable?

2. What is the equation of the line tangent to $f(x) = 2x^9 - 3x^2$ at the point where $x = 1$.

3. (The penalty for each mistake is five points.) Let

$$f(x) = \begin{cases} x+1 & \text{if } x \le 1, \\ x^2 - 1 & \text{if } 1 < x < 2, \\ -x+5 & \text{if } 2 \le x. \end{cases}$$

   (a) Graph $y = f(x)$.
   (b) Fill in the blanks:

$$f(1) = \_\_ \quad \lim_{x\to 1^+} f(x) = \_\_ \quad \lim_{x\to 1^-} f(x) = \_\_ \quad \lim_{x\to 1} f(x) = \_\_$$
$$f(2) = \_\_ \quad \lim_{x\to 2^+} f(x) = \_\_ \quad \lim_{x\to 2^-} f(x) = \_\_ \quad \lim_{x\to 2} f(x) = \_\_$$
$$f(3) = \_\_ \quad \lim_{x\to 3^+} f(x) = \_\_ \quad \lim_{x\to 3^-} f(x) = \_\_ \quad \lim_{x\to 3} f(x) = \_\_$$

   (c) Where is f discontinuous?
   (d) Where is f not differentiable?

4. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \sqrt{2x-1}$.

5. If $y = \dfrac{\sin(7x^2 + 3x^2 - 15x)}{(4x^5 + 5x^3 + 9x)^2}$, then find $\frac{dy}{dx}$.

6. Find $\frac{dy}{dx}$ for $6x^3y^2 + 2x = x\cos y$.

7. STATE both parts of the Fundamental Theorem of Calculus.

8. DEFINE the definite integral $\int_a^b f(x)\,dx$.

9. A 30-foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 4 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 6 feet from the wall?

10. Find $\displaystyle\int \left(\frac{2}{x^4} + \sqrt{2-3x}\,\right) dx$. Check your answer.

11. Find $\displaystyle\int x^2 \sin(8x^3+18)\,dx$. Check your answer.

12. Find $\displaystyle\int_0^1 \frac{x^2}{\sqrt{4x^3+18}}\,dx$.

13. Let
$$f(x) = 3x - x^3.$$
Find where $f(x)$ is increasing, decreasing, concave up, and concave down. Find the local extreme points and the points of inflection of $y = f(x)$. Find the vertical and horizontal asymptotes of $y = f(x)$. GRAPH $y = f(x)$.

14. Find the area of the region which is bounded by $y = x$, $y + x^2 = 0$ and $x = 2$.

15. Let R be the region in the first quadrant which is bounded by $y = x^2$, $x = 2$, and the $x-$ axis. Find the volume of the solid which is obtained by revolving R about the $x-$ axis.

16. Find the length of $y = \frac{2}{3}(x^2+1)^{3/2}$ from $x = 1$ to $x = 4$.

17. Find the area of the surface obtained by revolving $y = \sqrt{25 - x^2}$, from $x = -2$ to $x = 3$, about the $x-$ axis.

18. Let
$$f(x) = 16x^{-\frac{1}{3}} + x^{\frac{5}{3}}.$$
Find where $f(x)$ is increasing, decreasing, concave up, and concave down. Find the local extreme points and the points of inflection of $y = f(x)$. Find the vertical and horizontal asymptotes of $y = f(x)$. GRAPH $y = f(x)$.

19. An open box with a capacity of 72,000 cubic inches is needed. If the box must be twice as long as it is wide, what dimensions would require the least amount of material?