Recitation Time_____ PRINT your name ______________________________

Math 141, Exam 3, Spring 2009

The exam is worth a total of 50 points. There are 10 questions on 5 pages. Each problem is worth 5 points. **SHOW your work. Make your work be coherent and clear.** Write in complete sentences whenever this is possible. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

I will post the solutions on my website on Monday, March 9.

1. **Let** $y = \sin(\sqrt{2x^2 + \pi x})$ **. Find** $\frac{dy}{dx}$ **.**

We have

$$\boxed{y' = \frac{(4x + \pi)\cos(\sqrt{2x^2 + \pi x})}{2\sqrt{2x^2 + \pi x}}.}$$

2. **Let** $y = \ln(\sin(4x))$ **. Find** $\frac{dy}{dx}$ **.**

We have

$$\boxed{y' = \frac{4\cos(4x)}{\sin(4x)}.}$$

3. **Let** $4x^2y^3 + 9\cos y = 10x$ **. Find** $\frac{dy}{dx}$ **.**

Take $\frac{d}{dx}$ of both sides to get

$$12x^2y^2\frac{dy}{dx} + 8xy^3 - 9(\sin y)\frac{dy}{dx} = 10.$$

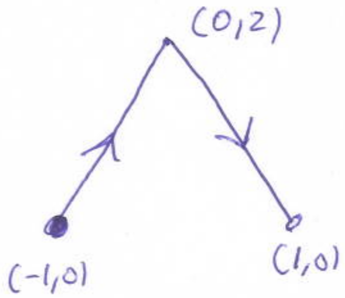
Solve for $\frac{dy}{dx}$ to see that

$$\boxed{\frac{dy}{dx} = \frac{10 - 8xy^3}{12x^2y^2 - 9\sin y}.}$$

4. **Find the equation of the line tangent to** $f(x) = x^{10} + 2x$ **at** $x = 1$ **.**

We want the line through $(1, f(1)) = (1, 3)$ with slope equal to $f'(1)$. We calculate $f'(x) = 10x^9 + 2$. Thus, $f'(1) = 12$ and the tangent line is $\boxed{y - 3 = 12(x - 1)}$.

5. **Parameterize the curve pictured below. Use t as your parameter with $0 \leq t \leq 2$. The point that corresponds to $t = 0$ is $(-1, 0)$. The point that corresponds to $t = 1$ is $(0, 2)$. The point that corresponds to $t = 2$ is $(1, 0)$. (Note: Each part of the curve that looks like a line segment is a line segment.)**



The first leg of the trip takes place on the line $y = 2x + 2$. For this leg, $x(t) = t - 1$ and $y(t) = 2(t-1) + 2$. The second leg of the trip takes place on the line $y(t) = -2x + 2$. For this leg of the trip, $x(t) = t - 1$ and $y(t) = -2(t-1) + 2$. Thus the trip is parameterized by

$$\boxed{\begin{array}{ll} x(t) = t - 1 & \text{for } 0 \leq t \leq 2 \\ y(t) = \begin{cases} 2t & \text{for } 0 \leq t \leq 1 \\ -2t + 4 & \text{for } 1 \leq t \leq 2 \end{cases} \end{array}}$$

6. **The height of an object above the ground is given by $y(t) = -16t^2 + 32t + 48$, where y is measured in feet and t is measured in seconds. Find the velocity of the object when it hits the ground. Be sure to give units.**

The object hits the ground when $y(t) = 0$; so, $-16(t^2 - 2t - 3) = 0$; so, $-16(t-3)(t+1) = 0$. The object hits the ground when $t = -1$ or $t = 3$. Of course, $t = -1$ is not relevant. The object hits the ground when $t = 3$. The velocity of the object is given by $v(t) = y'(t) = -32t + 32$. The velocity of the object when it hits the ground is $v(3) = -32(3) + 32 = \boxed{-64 \text{ ft/sec}}$.

7. **Compute** $\lim_{x\to\infty} \left(1 - \frac{2}{x}\right)^{3x}$.

We know that $\lim_{x\to\infty} \left(1 + \frac{r}{x}\right)^{x} = e^r$, for any constant r. We apply this formula with $r = -2$:

$$\lim_{x\to\infty} \left(1 - \frac{2}{x}\right)^{3x} = \lim_{x\to\infty} \left(\left(1 + \frac{-2}{x}\right)^{x}\right)^{3} = \left(e^{-2}\right)^3 = \boxed{e^{-6}}.$$

8. **Let** $f(x) = \sin(x)$. **Use the DEFINITION OF THE DERIVATIVE to find** $f'(x)$.

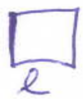
We have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} = \lim_{h\to 0} \left(\sin x \frac{\cos h - 1}{h} + \cos x \frac{\sin h}{h}\right).$$

We saw in class that $\lim_{h\to 0} \frac{\sin h}{h} = 1$. We also saw that $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$. We conclude that $f'(x) = \cos x$.

9. **Each side of a square is growing at the rate of 2 in/sec. How fast is the area of the square growing when each side has length 10 inches? Be sure to give units.**



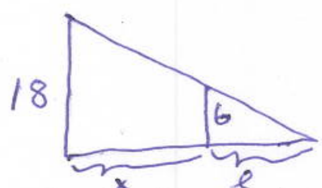
Let ℓ be the length of each side of the square, and A be the area of the square. We are told that $\frac{d\ell}{dt} = 2$ in/sec. We are to find $\frac{dA}{dt}|_{\ell=10\,\text{in}}$. We take $\frac{d}{dt}$ of both sides of $A = \ell^2$ to see that

$$\frac{dA}{dt} = 2\ell\frac{d\ell}{dt}.$$

Plug in to learn

$$\left.\frac{dA}{dt}\right|_{\ell=10\,\text{in}} = 2(10\,\text{in})(2\,\text{in/sec}) = \boxed{40\,\text{in}^2/\text{sec.}}$$

10. **A man 6 feet tall is walking away from an 18 foot tall street light at the rate of 3 ft/sec. At what rate is his shadow lengthening? Be sure to give units.**



Let x be the distance from the man to the light pole and ℓ be the length of his shadow. We are told $\frac{dx}{dt} = 3$ ft/sec. We want to find $\frac{d\ell}{dt}$. Use similar triangles to see that

$$\frac{\ell}{6} = \frac{\ell + x}{18}.$$

Clean the last equation up to see $3\ell = \ell + x$ or $2\ell = x$. Take the derivative to learn $2\frac{d\ell}{dt} = \frac{dx}{dt}$. We conclude that $\boxed{\frac{d\ell}{dt} = \frac{3}{2}\,\frac{\text{ft}}{\text{sec}}.}$