1999 Ex2 69

PRINT Your Name:__________________ Recitation Time __________

There are 10 problems on 4 pages. Each problem is worth 10 points. In one problem you are instructed to use the definition of the derivative; you MUST use the definition of the derivative in that problem. In the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. NO **CALCULATORS!**

1. The volume of a cube is growing at the rate of 6 cubic inches per second. Find the rate of change of the cube's surface area at the instant when each side has length 10 inches.

$V = s^3$

$A = 6s^2$

we know $\frac{dV}{dt} = 6$

We want $\left.\frac{dA}{dt}\right|_{s=10}$

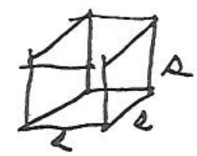
$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$

$\frac{\frac{dV}{dt}}{3s^2} = \frac{ds}{dt}$

$\frac{dA}{dt} = 12s\frac{ds}{dt}$

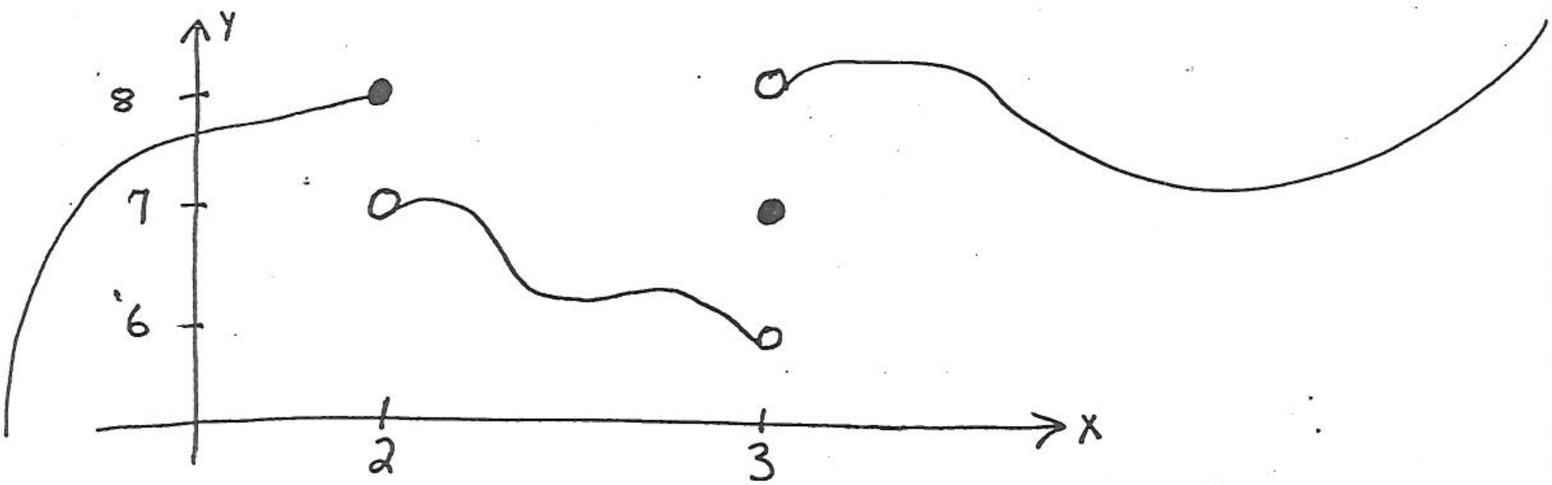
$\frac{dA}{dt} = 12s\,\frac{\frac{dV}{dt}}{3s^2} = \frac{4\frac{dV}{dt}}{s}$

$\left.\frac{dA}{dt}\right|_{s=10} = \frac{4\cdot 6}{10} = 2.4\ \frac{in^2}{sec.}$



There are 6 faces. Each face has area s^2.

2. **(The penalty for each mistake is five points.)** The picture represents the graph of $y = f(x)$.



Fill in the blanks:

$f(2) = 8$	$\lim_{x\to 2+} f(x) = 7$	$\lim_{x\to 2-} f(x) = 8$	$\lim_{x\to 2} f(x) =$ DNE
$f(3) = 7$	$\lim_{x\to 3+} f(x) = 8$	$\lim_{x\to 3-} f(x) = 6$	$\lim_{x\to 3} f(x) =$ DNE

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3. Find the maximum and the minimum of $f(x) = x^2 + 2x$ for $-2 \le x \le 1$.

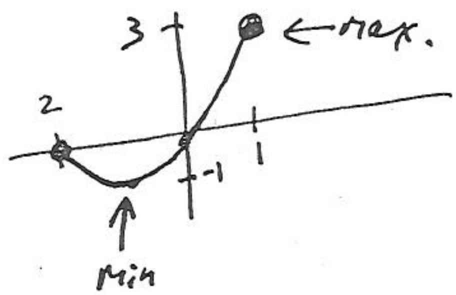
$f' = 2x + 2$

$f' = 0$ whe $x = -1$

$f(-1) = 1 - 2 = -1$

$f(-2) = 4 - 4 = 0$

$f(1) = 1 + 2 = 3$



The minimum is $(-1, -1)$

The maximum is $(1, 3)$

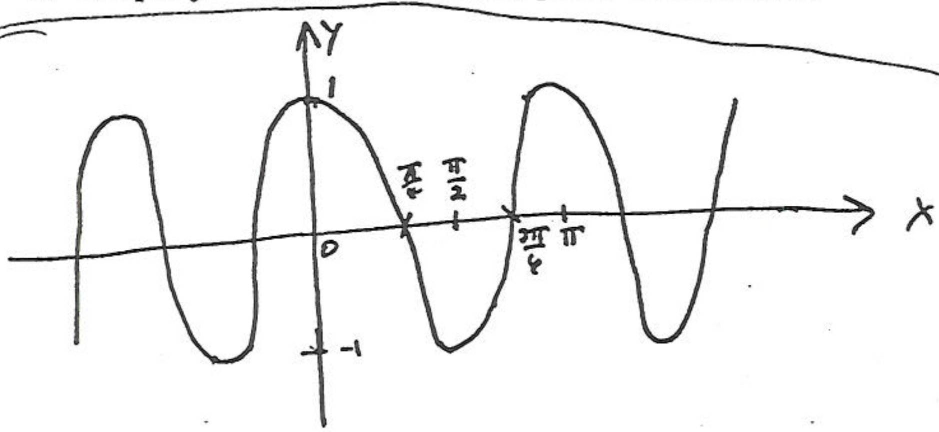
4. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \dfrac{1}{4x - 3}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{4(x+h)-3} - \frac{1}{4x-3}}{h} = \lim_{h \to 0} \frac{(4x-3) - (4(x+h)-3)}{(4(x+h)-3)(4x-3)h}$$

$$= \lim_{h \to 0} \frac{\cancel{4x} - \cancel{3} - (\cancel{4x} + 4h - \cancel{3})}{(4(x+h)-3)(4x-3)h} = \lim_{h \to 0} \frac{-4\cancel{h}}{(4(x+h)-3)(4x-3)\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-4}{(4(x+h)-3)(4x-3)} = \frac{-4}{(4x-3)(4x-3)} = \frac{-4}{(4x-3)^2}$$

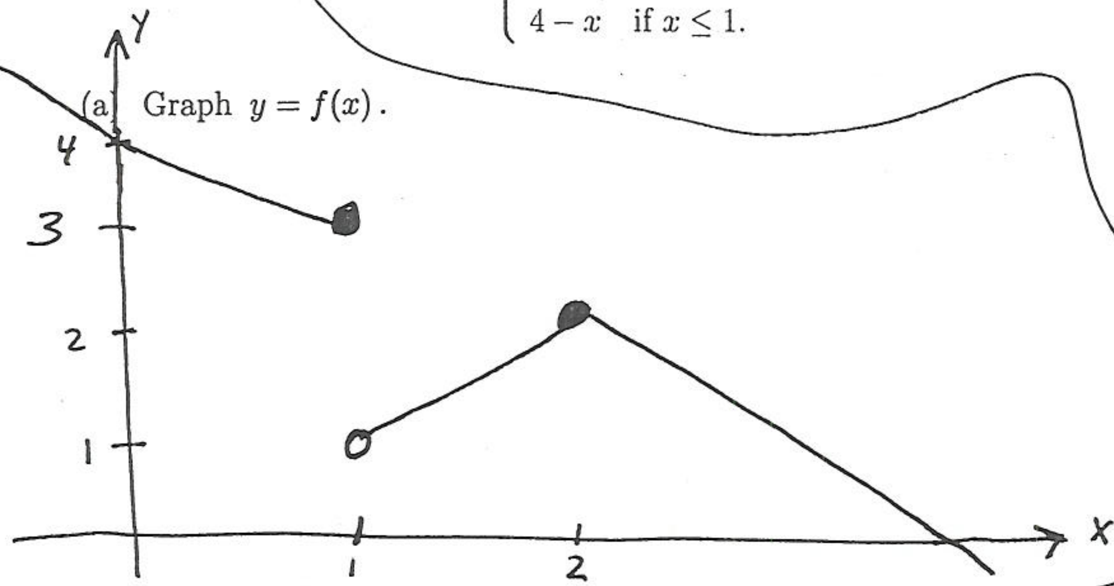
5. Graph $y = \cos 2x$. Mark a few points on each axis.



6. (The penalty for each mistake is five points.) Let

$$f(x) = \begin{cases} 4-x & \text{if } 2 \le x, \\ x & \text{if } 1 < x < 2, \text{ and} \\ 4-x & \text{if } x \le 1. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) =$ 4	$\lim_{x\to 0+} f(x) =$ 4	$\lim_{x\to 0^-} f(x) =$ 4	$\lim_{x\to 0} f(x) =$ 4
$f(1) =$ 3	$\lim_{x\to 1+} f(x) =$ 1	$\lim_{x\to 1^-} f(x) =$ 3	$\lim_{x\to 1} f(x) =$ DNE
$f(2) =$ 2	$\lim_{x\to 2+} f(x) =$ 2	$\lim_{x\to 2^-} f(x) =$ 2	$\lim_{x\to 2} f(x) =$ 2

(c) Where is $f(x)$ continuous?

f(x) is continuous for all x except $x = 1$

(d) Where is $f(x)$ differentiable?

f(x) is differentiable for all x except $x=1$ and $x=2$

7. Let $2x^3y^2 = \sin(2x^2y^4)$. Find $\frac{dy}{dx}$.

$$2x^3 2y\frac{dy}{dx} + 6x^2y^2 = \cos(2x^2y^4)\left[8x^2y^3\frac{dy}{dx} + 4xy^4\right]$$

$$\frac{dy}{dx}\left(4x^3y - 8x^2y^3\cos(2x^2y^4)\right] = 4xy^4\cos(2x^2y^4) - 6x^2y^2$$

$$\frac{dy}{dx} = \frac{4xy^4\cos(2x^2y^4) - 6x^2y^2}{4x^3y - 8x^2y^3\cos(2x^2y^4)}$$

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"x_1"

8. Find the equation of the line tangent to $f(x) = \sin^2 x$ at $x = \frac{\pi}{4}$.

$f(\frac{\pi}{4}) = [\sin(\frac{\pi}{4})]^2 = (\frac{\sqrt{2}}{2})^2 = \frac{2}{4} = \frac{1}{2}$

"y_1"

$f'(x) = 2 \sin x \cos x$

$f'(\frac{\pi}{4}) = 2 \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2} = \frac{2 \cdot 2}{2 \cdot 2} = 1$

"m"

$y - y_1 = m(x - x_1)$

$(y - \frac{1}{2}) = 1(x - \frac{\pi}{4})$

9. Let $y = \frac{x}{\sin x}$. Find dy.

$dy = \frac{\sin x - x\cos x}{\sin^2 x} dx$ (because $dy = \frac{dy}{dx} dx$)

10. Let $y = \sqrt{x^3 \cos^2(2x) + 19x^2}$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{38x + x^3 2\cos(2x)(-\sin(2x))2 + 3x^2\cos^2(2x)}{2\sqrt{x^3\cos^2(2x) + 19x^2}}$$