1995 Exam 2

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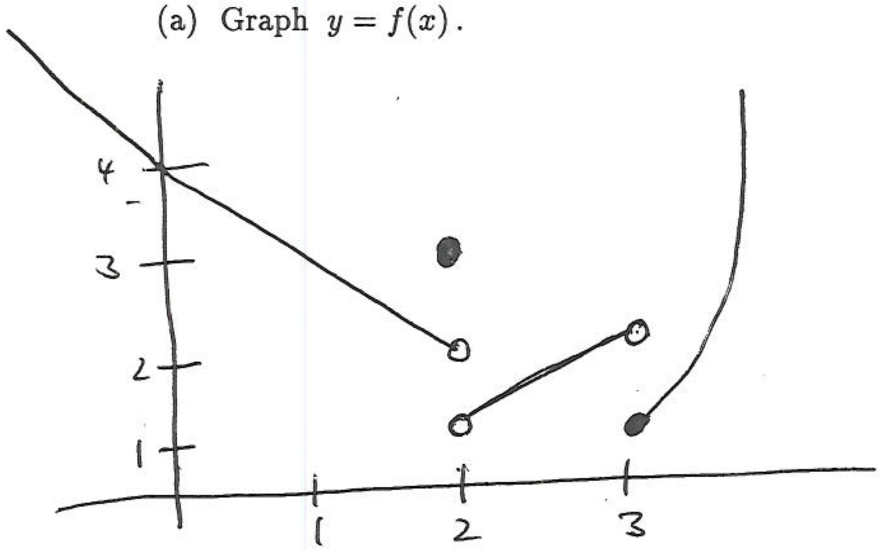
PRINT Your Name:______________________________

There are 13 problems on 6 pages. In problem 10 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{CIRCLE}$ your answer.

1. (10 points – The penalty for each mistake is four points.) Let

$$f(x) = \begin{cases} 4-x & \text{if } x < 2, \\ 3 & \text{if } x = 2, \\ x-1 & \text{if } 2 < x < 3, \text{ and} \\ x^2-8 & \text{if } 3 \le x. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(1) = 3$	$\lim_{x\to1^+} f(x) = 3$	$\lim_{x\to1^-} f(x) = 3$	$\lim_{x\to1} f(x) = 3$
$f(2) = 3$	$\lim_{x\to2^+} f(x) = 1$	$\lim_{x\to2^-} f(x) = 2$	$\lim_{x\to2} f(x) = DNE$
$f(3) = 1$	$\lim_{x\to3^+} f(x) = 1$	$\lim_{x\to3^-} f(x) = 2$	$\lim_{x\to3} f(x) = DNE$

2. (7 points) Let $y = \frac{1}{\sqrt{2x}} - \sin(2x)$. Find $\frac{dy}{dx}$.

$y = \frac{1}{\sqrt{2}} x^{-\frac{1}{2}} - \sin(2x)$

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right) x^{-\frac{3}{2}} - 2\cos(2x)$

$-(2x)^{-\frac{3}{2}} - 2\cos(2x)$

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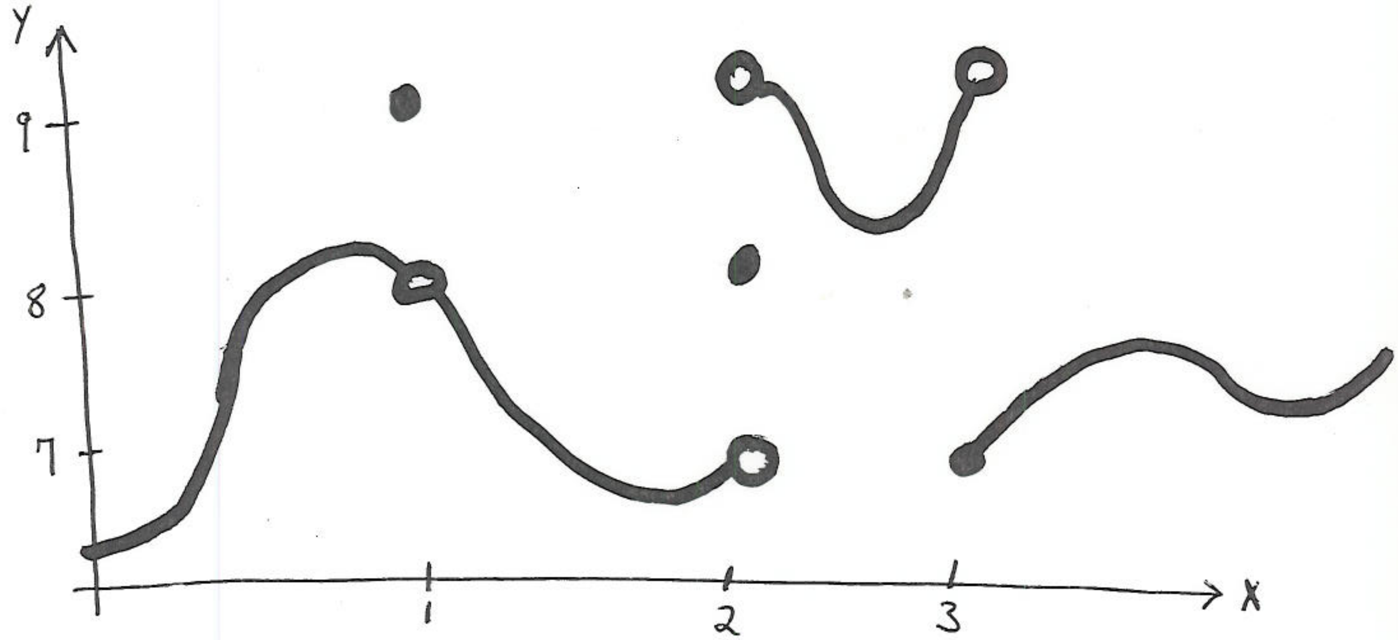
3. (10 points – The penalty for each mistake is four points.) The picture represents the graph of $y = f(x)$.



Fill in the blanks:

$f(1) = \underline{9}$ $\quad \lim_{x\to 1^+} f(x) = \underline{8}$ $\quad \lim_{x\to 1^-} f(x) = \underline{8}$ $\quad \lim_{x\to 1} f(x) = \underline{8}$

$f(2) = \underline{8}$ $\quad \lim_{x\to 2^+} f(x) = \underline{9}$ $\quad \lim_{x\to 2^-} f(x) = \underline{7}$ $\quad \lim_{x\to 2} f(x) = \underline{DNE}$

$f(3) = \underline{7}$ $\quad \lim_{x\to 3^+} f(x) = \underline{7}$ $\quad \lim_{x\to 3^-} f(x) = \underline{9}$ $\quad \lim_{x\to 3} f(x) = \underline{DNE}$

4. (7 points) Let $y = (2x^3 + \sqrt{2}x)^4(2x^5 + \cos(3x))^6$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 4(2x^3+\sqrt{2}\,x)^3(6x^2+\sqrt{2})(2x^5+\cos 3x)^6$$
$$+ (2x^3+\sqrt{2}\,x)^4\, 6(2x^5+\cos 3x)^5(10x^4-3\sin 3x)$$

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5. (7 points) Let $y = \dfrac{4x^5 + \frac{2}{x} + 19}{8x^3 + 15x + 6}$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{(8x^3+15x+6)(20x^4 - \frac{2}{x^2}) - (4x^5 + \frac{2}{x} + 19)(24x^2+15)}{(8x^3+15x+6)^2}$$

6. (7 points) Let $4xy^2 + \sin(xy) = 3y^2 + 6x^2$. Find $\frac{dy}{dx}$.

$$4x\,2y\frac{dy}{dx} + 4y^2 + \cos(xy)\left(x\frac{dy}{dx} + y\right) = 6y\frac{dy}{dx} + 12x$$

$$4y^2 + y\cos(xy) - 12x = \frac{dy}{dx}\left(6y - x\cos(xy) - 8xy\right)$$

$$\frac{4y^2 + y\cos(xy) - 12x}{6y - x\cos(xy) - 8xy} = \frac{dy}{dx}$$

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7. (7 points) Let $y = \sqrt{\sin^2(4x^2+3x+19)+\cos^3(x)}$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{2\sin(4x^2+3x+19)\cos(4x^2+3x+19)(8x+3) - 3\cos^2(x)\sin x}{2\sqrt{\sin^2(4x^2+3x+19)+\cos^3(x)}}$$

8. (8 points) A cube is growing at the constant rate of 1000 cubic inches per second. How fast is each edge growing when each edge is 5 inches long?

V = Vol of cube
ℓ = length of each edge

We know $\frac{dV}{dt} = 1000\ \text{in}^3/\text{sec}$

We want $\left.\frac{d\ell}{dt}\right|_{\ell=5\text{in}}$

$V = \ell^3$

$\frac{dV}{dt} = 3\ell^2\frac{d\ell}{dt}$

$\frac{d\ell}{dt} = \frac{1}{3\ell^2}\frac{dV}{dt}$

$\left.\frac{d\ell}{dt}\right|_{\ell=5} = \frac{1}{75}(1000)\ \text{in/sec}$

$\approx 13.3\overline{3}$ in/sec.

9. (7 points) Find the equation of the line tangent to $y = 3x^5+4x+2$ when $x=1$.

y-coord of the point is $y(1) = 3+4+2 = 9$

Pt is $(1,9)$

$y' = 15x^4+4$

slope is $y'(1) = 15+4 = 19$

$y-9 = 19(x-1)$

$y = 19x-10$

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10. (7 points) Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \sqrt{2x+1}$.

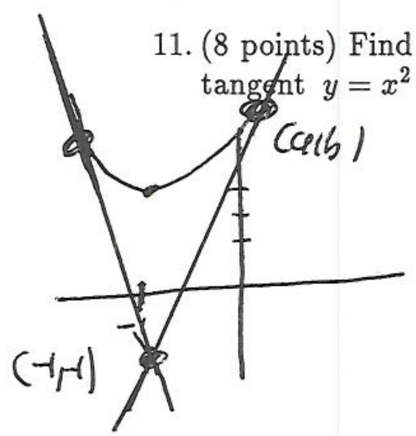
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{2x+2h+1} - \sqrt{2x+1}\right)\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x}+2h\cancel{+1} - (\cancel{2x+1})}{h\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)} = \lim_{h\to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h+1} + \sqrt{2x+1}\right)}$$

$$= \lim_{h\to 0} \frac{2}{\sqrt{2x+2h+1} + \sqrt{2x+1}} = \lim_{h\to 0} \frac{2}{\sqrt{2x+1} + \sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

11. (8 points) Find the equation of every line which passes through $(-1,-1)$ and is also tangent $y = x^2 + 2x + 4$.



I need all (a,b) on the parabola with $y'(a) =$ slope of line from $(-1,-1)$ to (a,b)

$b = a^2+2a+4$

$\dfrac{b+1}{a+1} = 2a+2$

$b+1 = (2a+2)(a+1)$

$a^2+2a+5 = (2a+2)(a+1)$

$a^2+2a+5 = 2a^2+4a+2$

$0 = a^2+2a-3$

$0 = (a+3)(a-1)$

$a = -3, 1$

when $a=-3$, $b=7$, then slope $= -6+2 = -4$ line is $\boxed{y+1 = -4(x+1)}$

when $a = 1$, then slope $= 2+2 = 4$ line is $\boxed{y+1 = 4(x+1)}$

$b=7$

$y = -4$

$y = 4x$

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12. (7 points) The position of an object above the earth's surface is given by

$$s(t) = -16t^2 + 48t + 64.$$

What is the velocity of the object when it strikes the ground?

the object hits the ground when $s(t)=0$, i.e. when

$0 = -16(t^2-3t-4) = -16(t-4)(t+1)$

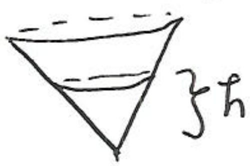
So $t = 4$ or -1

not interesting

ans $= v(4)$ $\quad v(t) = s'(t) = -32t+48$

$v(4) = -32(4)+48 = -80$ ft/sec

13. (8 points) A student is using a straw to drink from a conical cup, whose axis is vertical, at the rate of 3 cubic inches per second. If the height of the cup is 12 inches and the radius of its opening is 8 inches, how fast is the level of the liquid falling when the depth of the liquid is 7 inches? (Recall that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)



Let $V(t)$ = vol of liquid in the cup at time t

$h(t)$ = ht of liquid in the cup at time t

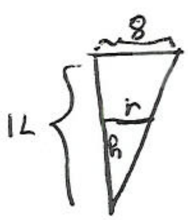
$r(t)$ = radius at the top of the liquid

we know $\frac{dV}{dt} = 3 \frac{in^3}{sec}$

we want $\left.\frac{dh}{dt}\right|_{h=7in}$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{4}{9} h^3$

$\frac{dV}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt}$



$\frac{r}{h} = \frac{8}{12}$

$\therefore r = \frac{2}{3}h$

$\frac{9}{4\pi h^2}\frac{dV}{dt} = \frac{dh}{dt}$

$\frac{9 \cdot 3}{4\pi 49}$ in/sec $= \left.\frac{dh}{dt}\right|_{h=7}$

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