1996 Exam 1

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PRINT Your Name:_____ Section:_____

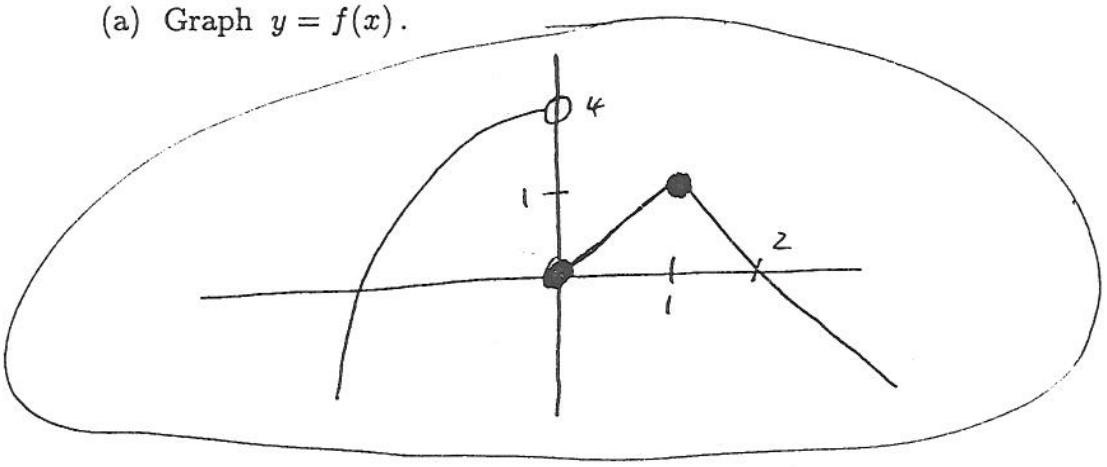
There are 8 problems on 3 pages. Problems 1 and 2 and are worth 20 points each. The other problems are worth 10 points each. In problem 3 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. CIRCLE your answer.

NO CALCULATORS!

1. **(The penalty for each mistake is five points.)** Let

$$f(x) = \begin{cases} 4 - x^2 & \text{if } x < 0, \\ x & \text{if } 0 \le x \le 1, \text{ and} \\ 2 - x & \text{if } 1 < x. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) = \underline{0}$	$\lim_{x\to 0^+} f(x) = \underline{0}$	$\lim_{x\to 0^-} f(x) = \underline{4}$	$\lim_{x\to 0} f(x) = \underline{DNE}$
$f(1) = \underline{1}$	$\lim_{x\to 1^+} f(x) = \underline{1}$	$\lim_{x\to 1^-} f(x) = \underline{1}$	$\lim_{x\to 1} f(x) = \underline{1}$
$f(2) = \underline{0}$	$\lim_{x\to 2^+} f(x) = \underline{0}$	$\lim_{x\to 2^-} f(x) = \underline{0}$	$\lim_{x\to 2} f(x) = \underline{0}$

(c) Where is $f(x)$ continuous? Every where except $X=0$.

(d) Where is $f(x)$ differentiable? Every where except $X=0$ and $X=1$.

2. Compute the following limits:

(a) $\lim_{x\to 3^+} \frac{x^2-9}{x-3} = \lim_{x\to 3^+} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3^+} x+3 = 6$

(c) $\lim_{x\to 3^+} \frac{x-3}{x^2-9} = \lim_{x\to 3^+} \frac{1}{x+3} = \frac{1}{6}$

(c) $\lim_{x\to 3^+} \frac{x^2-9}{x+3} = \lim_{x\to 3^+} x-3 = 0$

(d) $\lim_{x\to 3^+} \frac{x+3}{x^2-9} = \lim_{x\to 3^+} \frac{1}{x-3} = +\infty$

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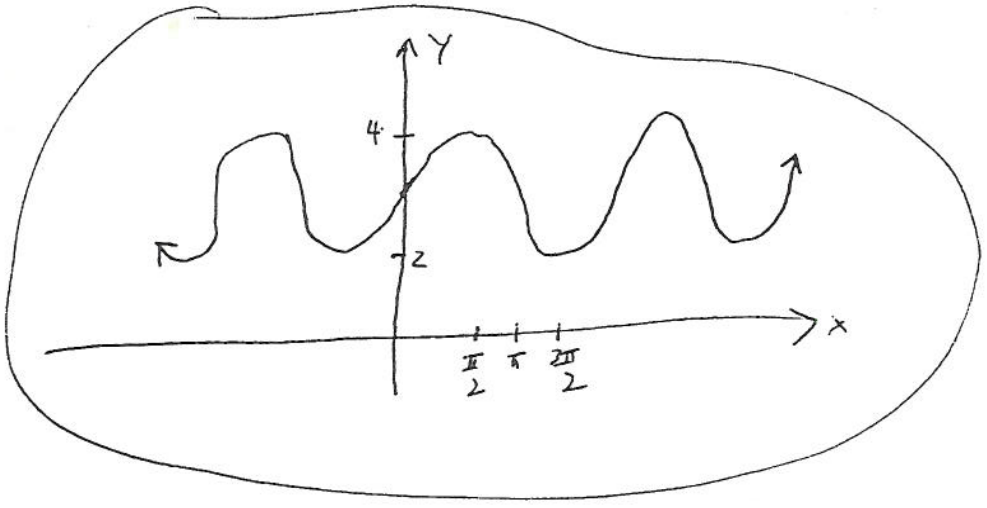
3. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = 4\sqrt{x-3}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{4\left(\sqrt{x+h-3} - \sqrt{x-3}\right)}{h}$$

$$= \lim_{h\to 0} \frac{4\left(\sqrt{x+h-3} - \sqrt{x-3}\right)\left(\sqrt{x+h-3} + \sqrt{x-3}\right)}{h\left(\sqrt{x+h-3} + \sqrt{x-3}\right)} = \lim_{h\to 0} \frac{4\left(x+h-3-(x-3)\right)}{h\left(\sqrt{x+h-3} + \sqrt{x-3}\right)}$$

$$= \lim_{h\to 0} \frac{4h}{h\left(\sqrt{x+h-3} + \sqrt{x-3}\right)} = \lim_{h\to 0} \frac{4}{\sqrt{x+h-3} + \sqrt{x-3}} = \frac{4}{2\sqrt{x-3}} = \boxed{\frac{2}{\sqrt{x-3}}}$$

4. Graph $y = 3 + \sin x$.



5. Find the equation of the line tangent to $f(x) = 9x^{10} + 8x$ at $x = -1$.

$f(-1) = 9 - 8 = 1 \quad f'(x) = 90x^9 + 8 \quad f'(-1) = -90 + 8 = -82$

$$\boxed{(y-1) = -82(x+1)}$$

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6. Let $f(x) = x^2 \cos x$. Find $f'(x)$.

$f'(x) = -x^2 \sin x + 2x \cos x$

7. Let $f(x) = \dfrac{x^3 + 9x}{\sin x}$. Find $f'(x)$.

$f'(x) = \dfrac{\sin x\,(3x^2+9) - (x^3+9x)\cos x}{\sin^2 x}$

8. Let $f(x) = 9x^3 + \dfrac{9}{x} + 4\sqrt{x} + 16$. Find $f'(x)$.

$f'(x) = 27x^2 - \dfrac{9}{x^2} + \dfrac{2}{\sqrt{x}}$

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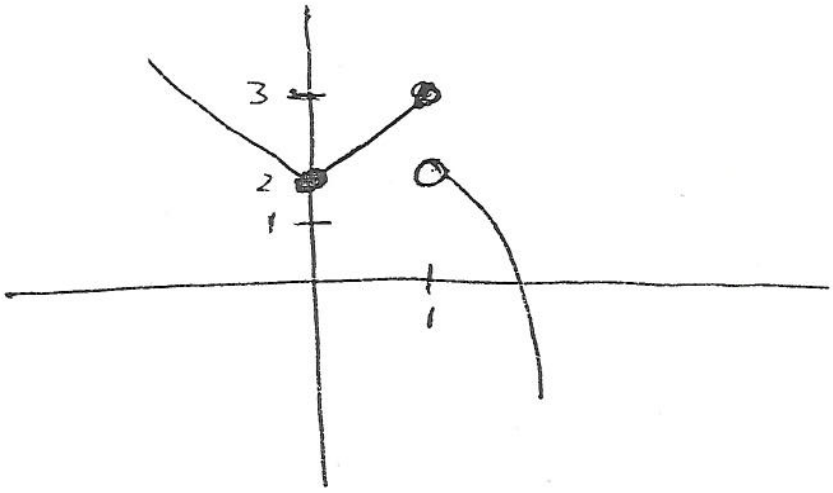
There are 10 problems on 5 pages. Each problems is worth 10 point. In problem 2 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. CIRCLE your answer.
NO CALCULATORS!

1. **(The penalty for each mistake is five points.)** Let

$$f(x) = \begin{cases} 2 - x & \text{if } x < 0, \\ 2 + x & \text{if } 0 \le x \le 1, \text{ and} \\ 3 - x^2 & \text{if } 1 < x. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) = 2$	$\lim_{x \to 0^+} f(x) = 2$	$\lim_{x \to 0^-} f(x) = 2$	$\lim_{x \to 0} f(x) = 2$
$f(1) = 3$	$\lim_{x \to 1^+} f(x) = 0$	$\lim_{x \to 1^-} f(x) = 3$	$\lim_{x \to 1} f(x) = DNE$
$f(2) = -1$	$\lim_{x \to 2^+} f(x) = -1$	$\lim_{x \to 2^-} f(x) = -1$	$\lim_{x \to 2} f(x) = -1$

(c) Where is $f(x)$ continuous?

Everywhere except $x = 1$.

(d) Where is $f(x)$ differentiable?

Everywhere except $x = 0$ and $x = 1$.