Math 141, Exam 4, 2000

PRINT Your Name:_____

Recitation Time: _____ Tu. Th.

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** You might find the following formulas to be useful:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

1. STATE both parts of the Fundamental Theorem of Calculus.

2. Find $\int x \sin(x^2+4)dx$. Be sure to check your answer.

3. DEFINE the definite integral $\int_a^b f(x)\,dx$.

4. A 20-foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 5 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 7 feet from the wall?

5. Let $y = x\cos^3(4x^2+3) + \sin^4(x)$. Find $\frac{dy}{dx}$.

6. Let $f(x) = 8x^2 - x^4$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find all local maximum points, local minimum points, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

7. Solve the Initial Value Problem $\frac{dy}{dx} = y^4$, $y(1) = \frac{1}{2}$. Be sure to check your answer.

8. Find the area of the region between $y = 2 - x^2$ and $y = x$.

9. Consider the region bounded by $y = x^2$, $y = 0$, $x = 2$ and $x = 4$. Rotate this region about the x-axis. Find the volume of the resulting solid.

10. Consider the region A, which is bounded by the $x-$ axis, $y = x^2$, $x = 0$, and $x = 2$. Consider 100 rectangles, all with base $1/50$, which UNDER estimate the area of A. How much area is inside the 100 rectangles? (You must answer the question I asked. I expect an exact answer in closed form.)