Title: The bi-graded structure of Symmetric Algebras with applications to Rees rings.

Abstract: Let *k* be a field, R = k[x, y] a polynomial ring in 2 variables over *k*, and *I* a height 2 ideal of *R* minimally generated by 3 forms, g_1, g_2, g_3 of the same positive degree *d*. The Hilbert-Burch Theorem guarantees that there is a 3×2 matrix φ , with homogeneous entries from *R*, so that the signed 2×2 minors of φ are equal to g_1, g_2 , and g_3 . We arrange φ so that each entry in column *i* of φ has degree d_i , with $d_1 < d_2$. Let \mathcal{R} be the Rees algebra of *I*, that is

$$\mathcal{R} = R \oplus I \oplus I^2 \oplus I^3 \oplus \cdots = R[It],$$

 \mathcal{A} be the kernel of the natural surjection

$$\operatorname{Sym}(I) \twoheadrightarrow \mathcal{R}$$

from the symmetric algebra of *I* to the Rees algebra of *I* and let *S* and *B* be the polynomial rings $S = k[T_1, T_2, T_3]$ and $B = R \otimes_k S = k[x, y, T_1, T_2, T_3]$. View *B* as a bi-graded *k*-algebra, where *x* and *y* have bi-degree (1,0) and each T_i has bi-degree (0,1). We describe the *S*-module structure of $\mathcal{A}_{(\geq d_1-1,*)}$ under the hypothesis that φ has a generalized zero in its first column. This module is free and we identify the bi-degrees of its basis. We also identify the bi-degrees of a minimal generating set of $\mathcal{A}_{(\geq d_1-1,*)}$ as an ideal of Sym(*I*). Our proof is motivated by a Theorem of Weierstrass and Kronecker which classifies matrices with homogeneous linear entries in two variables. When one views this result in a geometric context, that is,

$$[g_1,g_2,g_3]: \mathbb{P}^1 \to \mathcal{C} \subseteq \mathbb{P}^2$$

is a birational parameterization of a plane curve, then Bi-Proj \mathcal{R} is the graph, Γ , of the parameterization of C, the hypothesis concerning the existence of a generalized zero is equivalent to assuming that C has a singularity of multiplicity d_2 , and the ideal $\mathcal{A}_{(\geq d_1-1,*)}$ is an approximation of the ideal \mathcal{A} which defines Γ .