

Notes on piecewise smooth elliptic boundary value problems

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Introduction

These notes are concerned with the theory of elliptic boundary value problems on a piecewise smooth plane domain, and with piecewise smooth coefficients. The immediate purpose is to give a simple account, as self contained as possible, of those portions of the theory that are of use in the numerical and asymptotic analysis of these problems. The techniques employed were developed in the seminal paper of Kondrat'yev [1967], and have already been carefully expounded by Grisvard [1985, 1992], Dauge [1988], Nazarov and Plamenevskii [1994] and Kozlov, Maz'ya and Rossmann [1997].

A decimal numbering system is used for cross-referencing, with the number after the semicolon referring to the item in the subsection. Thus, (1.2;1) and Lemma 1.2.1;1 refer to equation (1) of §1.2 and to Lemma 1 of §1.2.1 respectively. Each section is dated. New sections, or new versions of existing sections will be generated when appropriate.

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