7.3 **#8** $\mathcal{L}\{e^{-2t}\cos 4t\}$

$\mathcal{L}\{\cos 4t\} = \dfrac{s}{s^2+16}$. Therefore, by the first translation theorem

$$\mathcal{L}\{e^{-2t}\cos 4t\} = \frac{s-(-2)}{[s-(-2)]^2+16} = \boxed{\frac{s+2}{(s+2)^2+16}}$$

7.3 **#18** $\mathcal{L}^{-1}\left\{\dfrac{5s}{(s-2)^2}\right\}$

Observe that shifting so that $(s-2)$ is replaced by s will result in a function whose inverse Laplace transform is easier to find. Thus, using the inverse version of the first translation theorem, we obtain:

$$\mathcal{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\} = e^{2t}\mathcal{L}^{-1}\left\{\frac{5(s+2)}{[(s+2)-2]^2}\right\} = 5e^{2t}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2}\right\} =$$

$$= 5e^{2t}\left(\mathcal{L}^{-1}\left\{\frac{\cancel{s}^{\,1}}{\cancel{s^2}_{\,s}}\right\} + \mathcal{L}^{-1}\left\{\frac{2}{s^2}\right\}\right) = \boxed{5e^{2t}(1+2t)}$$

7.3 **#40**

$$\mathcal{L}\{(3t+1)\mathcal{U}(t-1)\} \overset{\text{using the second translation theorem}}{=} e^{-s}\mathcal{L}\{3(t+1)+1\} = e^{-s}\mathcal{L}\{3t+4\} =$$

$$= e^{-s}\left(3\cdot\frac{1}{s^2} + 4\cdot\frac{1}{s}\right) = \boxed{e^{-s}\left(\frac{3}{s^2}+\frac{4}{s}\right)}$$

7.3 **#48**

$$\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\} = \mathcal{L}^{-1}\left\{e^{-2\cdot s}\cdot\underbrace{\frac{1}{s^2(s-1)}}_{F(s)}\right\} = f(t-2)\cdot\mathcal{U}(t-2)$$

where $f(t) = \mathcal{L}^{-1}\{F(s)\} =$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\} =$$

$$= \mathcal{L}^{-1}\left\{-\frac{1}{s} - \frac{1}{s^2} + \frac{1}{(s-1)}\right\} = -1 - t + e^t$$

$$\frac{1}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1}$$

$$1 = As^2 - As + Bs - B + Cs^2$$

$$\begin{cases} A + C = 0 \\ -A + B = 0 \\ -B = 1 \end{cases}$$

So $B = -1$, $A = B = -1$, $C = -A = +1$

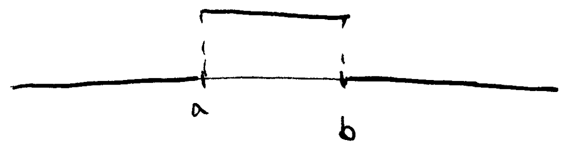
#48 cont.

So $\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s^2(s-1)}\right\} = \left(-1-(t-2)+e^{t-2}\right)\mathcal{U}(t-2) =$

$$= \boxed{\left(e^{t-2}-t+1\right)\mathcal{U}(t-2) = \begin{cases} 0 & \text{if } t \le 2 \\ e^{t-2}-t+1 & \text{if } t > 2 \end{cases}}$$

7.3 #56

$$f(t) = \begin{cases} 1 & 0 \le t < 4 \\ 0 & 4 \le t < 5 \\ 1 & 5 \le t \end{cases}$$



$\mathcal{U}(t-a) - \mathcal{U}(t-b)$

$$f(t) = \left[1-\mathcal{U}(t-4)\right]\cdot 1 + \left[\mathcal{U}(t-4)-\mathcal{U}(t-5)\right]\cdot 0 + \mathcal{U}(t-5)\cdot 1 =$$

$$= 1 - \mathcal{U}(t-4) + \mathcal{U}(t-5)$$

$$\mathcal{L}\{1-\mathcal{U}(t-4)+\mathcal{U}(t-5)\} = \mathcal{L}\{1\} - \mathcal{L}\{\mathcal{U}(t-4)\} + \mathcal{L}\{\mathcal{U}(t-5)\} =$$

$$= \frac{1}{s} - \frac{e^{-4s}}{s} + \frac{e^{-5s}}{s} = \boxed{\frac{1}{s}\left(1-e^{-4s}+e^{-5s}\right)}$$

$\boxed{7.3}$ #70 $\quad y'' + 4y' + 3y = 1 - \mathcal{U}(t-2) - \mathcal{U}(t-4) + \mathcal{U}(t-6) \qquad y(0)=0, \; y'(0)=0$

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 3\mathcal{L}\{y\} = \mathcal{L}\{1\} - \mathcal{L}\{\mathcal{U}(t-2)\} - \mathcal{L}\{\mathcal{U}(t-4)\} + \mathcal{L}\{\mathcal{U}(t-6)\}$$

$$\left(s^2 \cdot Y(s) - s\underset{0}{\underset{\shortparallel}{y(0)}} - \underset{0}{\underset{\shortparallel}{y'(0)}}\right) + 4\left(s\,Y(s) - \underset{0}{\underset{\shortparallel}{y(0)}}\right) + 3Y(s) = \frac{1}{s} - \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s} + \frac{e^{-6s}}{s}$$

$$Y(s)\left(s^2+4s+3\right) = \frac{1}{s}\left(1 - e^{-2s} - e^{-4s} + e^{-6s}\right)$$

$$Y(s)(s+1)(s+3) = \frac{1}{s}\left(1 - e^{-2s} - e^{-4s} + e^{-6s}\right)$$

$$Y(s) = \frac{1}{s(s+1)(s+3)}\left(1 - e^{-2s} - e^{-4s} + e^{-6s}\right)$$

$$\frac{1}{s(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}$$

$$1 = A(s+1)(s+3) + Bs(s+3) + Cs(s+1)$$

if $s=0$ we get : $1 = A\cdot 1\cdot 3 + 0 + 0$. So $\quad A = \frac{1}{3}$

if $s=-1$ we get : $1 = 0 + B\cdot(-1)\cdot 2 + 0$. So $\quad B = -\frac{1}{2}$

if $s=-3$ we get : $1 = 0 + 0 + C\cdot(-3)\cdot(-2)$. So $\quad C = \frac{1}{6}$

Thus $Y(s) = \left(\frac{1}{3}\cdot\frac{1}{s} - \frac{1}{2}\cdot\frac{1}{(s+1)} + \frac{1}{6}\cdot\frac{1}{(s+3)}\right)\left(1 - e^{-2s} - e^{-4s} + e^{-6s}\right).$

So $y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s}\right\} + \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s}\right\}$

$- \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s+1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s+1}\right\} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s+1}\right\}$

$+ \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s+3}\right\} - \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s+3}\right\} + \frac{1}{6}\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{s+3}\right\} =$