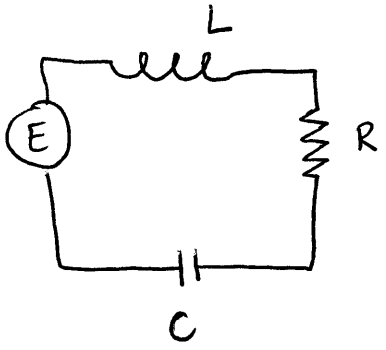
5.1 #48



$$Lq'' + Rq' + \frac{1}{C}q = E(t) \qquad q(0) = 0, \quad i(0) = q'(0) = 2$$

$$q'' + 100q' + \frac{1}{0.0004}q = 30$$

$$q'' + 100q' + 2500q = 30$$

1) $t^2 + 100t + 2500 = 0$

$(t+50)^2 = 0 \qquad t = -50$ mult. 2

$$q_{hom} = C_1 e^{-50t} + C_2 t e^{-50t}$$

2) $q_p = A$

$q_p' = 0$

$q_p'' = 0$

$2500A = 30$

$A = \frac{3}{250} = \frac{12}{1000} = 0.012$

So $q(t) = C_1 e^{-50t} + C_2 t e^{-50t} + 0.012$

$0 = q(0) = C_1 + 0 + 0.012 \qquad C_1 = -0.012$

$q'(t) = -50C_1 e^{-50t} + C_2 e^{-50t} - 50C_2 t e^{-50t}$

$2 = q'(0) = -50C_1 + C_2 - 0 \qquad C_2 = 2 + 50C_1 = 2 + 50(-0.012) = 2 - 0.6 = 1.4$

$$q(t) = -0.012 e^{-50t} + 1.4 t e^{-50t} + 0.012 = \boxed{e^{-50t}(1.4t - 0.012) + 0.012}$$

$q'(t) = e^{-50t}(-50(1.4t - 0.012) + 1.4) = e^{-50t}(-70t + 2)$

$q'(t) = 0$ iff $-70t + 2 = 0$ i.e. $t = \frac{2}{70} = \frac{1}{35}$

t	0	$\frac{1}{35}$	$+\infty$
$q'(t)$	+	0	−
$q(t)$	↗	max	↘

$q_{max} = q\left(\frac{1}{35}\right) = e^{-\frac{50}{35}}(0.04 - 0.012) + 0.012 =$

$= \boxed{0.0187 C}$

5.1 #52

$$\frac{1}{2}q'' + 20q' + \frac{1}{0.001}q = 100\sin(60t) + 200\cos(60t)$$

$$\frac{1}{2}q'' + 40q' + 2000q = 200\sin(60t) + 400\cos(60t)$$

1) $r^2 + 40r + 2000 = 0$

$$r_{1,2} = \frac{-40 \pm \sqrt{1600 - 8000}}{2} = \frac{-40 \pm \sqrt{-6400}}{2} = \frac{-40 \pm 80i}{2} = -20 \pm 40i$$

$$q_h = c_1 e^{-20t}\cos(40t) + c_2 e^{-20t}\sin(40t)$$

2) $q_{p_1} = A\sin(60t) + B\cos(60t)$

$$q'_{p_1} = 60A\cos(60t) - 60B\sin(60t)$$

$$q''_{p_1} = -3600A\sin(60t) - 3600B\cos(60t)$$

$$-3600A\sin(60t) - 3600B\cos(60t) + 40\left(60A\cos(60t) - 60B\sin(60t)\right) + 2000\left(A\sin(60t) + B\cos(60t)\right) = 200\sin(60t)$$

$$\begin{cases} -3600B + 2400A + 2000B = 0 \\ -3600A - 2400B + 2000A = 200 \end{cases}$$

$$\begin{cases} 2400A - 1600B = 0 & \div 800 \\ -1600A - 2400B = 200 & \div 200 \end{cases}$$

$$\begin{cases} 3A - 2B = 0 \\ -8A - 12B = 1 \end{cases}$$

$B = \frac{3}{2}A$

$-8A - 12 \cdot \frac{3}{2}A = 1$

$-8A - 18A = 1$

$-26A = 1$

$A = -\frac{1}{26}$ $\quad B = -\frac{3}{52}$

$$q_{p_1}(t) = -\frac{1}{26}\sin(60t) - \frac{3}{52}\cos(60t)$$

(#52) cont.

3) $q_{p_2} = C\cos 40t + D\sin 40t$

$q'_{p_2} = -40C\sin 40t + 40D\cos 40t$

$q''_{p_2} = -1600C\cos 40t - 1600D\sin 40t$

$-1600C\cos 40t - 1600D\sin 40t + 40(-40C\sin 40t + 40D\cos 40t)$

$+2000(C\cos 40t + D\sin 40t) = 400\cos 40t$

$$\begin{vmatrix} -1600C + 1600D + 2000C = 400 \\ -1600D - 1600C + 2000D = 0 \end{vmatrix}$$

$$\begin{vmatrix} 400C + 1600D = 400 & 1 \div 400 \\ -1600C + 400D = 0 & 1 \div 400 \end{vmatrix}$$

$$\begin{vmatrix} C + 4D = 1 \\ -4C + D = 0 \end{vmatrix} \qquad D = 4C \qquad C + 16C = 1$$

$$\boxed{C = \frac{1}{17}} \qquad \boxed{D = \frac{4}{17}}$$

$$q_{p_2}(t) = \frac{1}{17}\cos 40t + \frac{4}{17}\sin 40t$$

So $q_p(t) = -\frac{1}{26}\sin(60t) - \frac{3}{52}\cos(60t) + \frac{4}{17}\sin(40t) + \frac{1}{17}\cos(40t)$

The steady-state current is

$$\boxed{i_p(t) = q'_p(t) = -\frac{60}{26}\cos(60t) + \frac{180}{52}\sin(60t) + \frac{160}{17}\cos(40t) - \frac{40}{17}\sin(40t)}$$