SOLUTIN TO SAMPLE PROBLEMS - TEST 3 MATH 242

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Here are the solutions to the sample problems. If you get a different answer, check with your classmates and verify your results. Hopefully, these answers are correct, but sometimes there are errors from incorrect typing. Good luck on your test.

1. Laplace Transforms

Know the different rules. Know the basic transforms for the polynomials and trigonometric functions. Then there are other rules that are used when you have a function multiplied by an exponential function and the heavyside function (see problem 1f). Lots of rules that you have to know. Learn them !!

1a. $t + \cos 3t \quad \rightarrow \quad \frac{1}{s^2} + \frac{s}{s^2+9}$

1b. $te^{-t} \quad \rightarrow \quad \frac{1}{(s+1)^2}$

1c. $3\sin 2t + e^{-t} \quad \rightarrow \quad \frac{6}{s^2+4} + \frac{1}{t+1}$

1d. $\cos^2 3t \quad \rightarrow \quad \frac{s^2+18}{s(s^2+36)}$

1e. $t^2 \sin t \quad \rightarrow \quad (-1)^2 \frac{d^2}{ds^2}[\frac{1}{s^2+1}] = \frac{d}{ds}[\frac{-2s}{(s^2+1)^2}] = -\frac{2(s+1)^2 - 2s \cdot 2(s^2+1)\cdot 2s}{(s^2+1)^4}$

1f. $U(t-1)e^{t-2} = U(t-1)e^{(t-1)-1} \quad \rightarrow \quad e^{-s}L\{e^{t-1}\} = \frac{e^{-s}}{e}\frac{1}{s-1}$

1f. $t * \sin 3t \quad \rightarrow \quad L\{t\}L\{\sin 3t\} = \frac{1}{s^2}\frac{3}{s^2+9}$

2. Inverse Transforms

First, know the tranform rules. Also make sure you know how to do partial fractions.

2a. $\frac{1}{(s-5)^2} \quad \rightarrow \quad e^{5t}t$

2b. $\frac{s+5}{s^2-6s+13} = \frac{s+5}{(s-3)+4} = \frac{s-3}{(s-3)+4} + \frac{8}{(s-3)+4} = \quad \rightarrow \quad e^{3t}\cos 2t + 4e^{3t}\sin 2t$

2c. $\frac{s+5}{s^2+6s+13} = \frac{s+5}{(s+3)+4} = \frac{s+3}{(s+3)+4} + \frac{2}{(s-3)+4} = \quad \rightarrow \quad e^{-3t}\cos 2t + e^{-3t}\sin 2t$

2d. $\frac{s+2}{s^2-1} = \frac{s+2}{(s-1)(s+1)} = \frac{A}{s-1} + \frac{B}{s+1} = \frac{(A+B)s+(A-B)}{(s-1)(s+1)}$

Solve $A + B = 1$ and $A - B = 2$. You get $A = 1.5$ and $B = -0.5$.

$$\frac{A}{s-1} + \frac{B}{s+1} \quad \rightarrow \quad 1.5e^t - 0.5e^{-t}$$

2e. $\frac{e^{-s}}{s(s+1)} = e^{-s}[\frac{1}{s} - \frac{1}{s+1}] = e^{-s}\frac{1}{s} - e^{-s}\frac{1}{s+1} \quad \rightarrow \quad U(t-1)\cdot 1 - U(t-1)e^{-(t-1)}$

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3. Initial Value Problems

You have to know the rule for taking the Laplace transform of a derivative. The Laplace of a derivative (first, second, third, ...) uses the initial values. After you take the Laplace of the given differential equation, solve for Y which is the Laplacian of your solution y, then take the inverse Laplace tranform of Y.
3a. Notice this is not the same problem as on your sheet.

$$y'' - 4y' + 4y = t^3 e^{2t} \quad y(0) = 0 \quad y'(0) = 0$$

$$(s^2 - 4s + 4)Y = \frac{6}{(s-2)^4}$$

$$Y = \frac{6}{(s-2)^6} = \frac{6}{5!}\frac{5!}{(s-2)^6} \quad \rightarrow \quad y(t) = \frac{1}{20}t^5 e2t$$

3b.

$$y'' - 4y = t \quad y(0) = 1 \quad y'(0) = 0$$

$$(s^2 - 4)Y = s + \frac{1}{s^2} = \frac{s^3+1}{s^2}$$

From this, you have to setup the partial fractions problem.

$$\frac{s^3+1}{s^2(s^2-4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-2} + \frac{D}{s+2}$$

where $A = 0$, $B = -0.25$, $C = 9/16$, and $D = 7/16$. Hopefully, these are correct, but whatever the coefficients are the final answer will be:

$$y(t) = A + Bt + Ce^{2t} + De^{-2t}$$

3c.

$$y'' + y = \sin t \quad y(0) = 1 \quad y'(0) = -1$$

$$(s^2 + 1)Y = s - 1 + \frac{1}{s^2+1}$$

$$Y = \frac{s}{(s^2+1)} - \frac{1}{s^2+1} + \frac{1}{(s^2+1)^2}$$

From this, apply the rules to take the inverse.

$$y(t) = \cos t - \sin t + \frac{1}{2}(\sin t - t\cos t)$$

which could be simplified to the following.

$$y(t) = \cos t - \frac{1}{2}\sin t - \frac{1}{2}t\cos t$$