Study Guide

Thanksgiving 2003

Hope everyone has a pleasant Thanksgiving weekend. This homework assignment will get you off on the right foot for studying for the final exam. This will be due on December 1. Write each problem up so you can use your work as a review sheet for the rest of the week before the final exam.

Our exam will be held on December 8 at 9:00 in our same meeting place.

Show all work. Work should be written in an orderly manner with simple descriptions of what expressions or equations mean so anyone can understand the logical flow of your arguments.

Simplify into sines and cosines: $\tan(3x)\sec(4x)\csc(5x)\cot(6x)$

$$\frac{\sin 3x}{\cos 3x}\cdot\frac{1}{\cos 4x}\cdot\frac{1}{\sin 5x}\cdot\frac{\cos 6x}{\sin 6x}$$

Evaluate: $\lim_{x->0}\sin(4x)\csc(8x)$

$$L=\lim_{x\to 0}\frac{\sin 4x}{\sin 8x}\cdot\frac{8x}{4x\cdot 2}=\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{8x}{\sin 8x}\cdot\frac{1}{2}=\frac{1}{2}$$

Evaluate: $\lim_{x->0}\tan(4x)\cot(5x)$

$$L=\lim_{x\to 0}\frac{\sin 4x}{\cos 4x}\cdot\frac{\cos 5x}{\sin 5x}\cdot\frac{4\cdot 5x}{5\cdot 4x}=\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{5x}{\sin 5x}\cdot\frac{4}{5}\cdot\frac{\cos 5x}{\cos 4x}$$

$$=\frac{4}{5}$$

Evaluate: $\lim_{x->2} \frac{\sin(x^2-6x+8)}{x^2+4x-12}$

$$L = \lim_{x \to 2} \frac{\sin((x-2)(x-4))}{(x-2)(x+6)} \cdot \frac{x-4}{x-4} = \lim_{x \to 2} \frac{\sin((x-2)(x-4))}{(x-2)(x-4)} \cdot \frac{x-4}{x+6} = \frac{-2}{8} =$$

Evaluate: $\lim_{x->1} \frac{x^3-5x+4}{x^2+3x-4}$

$$L = \lim_{x \to 1} \frac{(x-1)(x^2+x-4)}{(x-1)(x+4)} = \lim_{x \to 1} \frac{x^2+x-4}{x+4} = -\frac{2}{5}$$

Evaluate: $\lim_{x->1} \frac{x^3-9x+4}{3x^2+3x-4}$

$$L = \frac{1-9+4}{3+3-4} = \frac{-4}{2} = -2$$

Evaluate: $\lim_{x->\infty} \frac{x^3-9x+4}{3x^4+3x-4}$

$$L = \lim_{x \to \infty} \frac{x^3-9x+4}{3x^4+3x-4} \frac{\left(\frac{1}{x^4}\right)}{\left(\frac{1}{x^4}\right)} = \lim_{x \to \infty} \frac{\frac{1}{x} - \frac{9}{x^3} + 4/x^4}{3 + 3/x^3 - 4/x^4} = \frac{0}{3} = 0$$

Evaluate: $\lim_{x->\infty} \frac{7x^2-9x+4}{x^2+3x-4}$

$$L = \lim_{x \to \infty} \frac{7x^2-9x+4}{x^2+3x-4} \frac{\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)} = \lim_{x \to \infty} \frac{7 - 9/x + 4/x^2}{1 + 3/x - 4/x^2} = \frac{7}{1} = 7$$

Is the function $y = \frac{\sin 3x}{4x}$ well-defined at $x = 0$? Why is it important to know if the limit $\lim_{x->0} \frac{\sin 3x}{4x}$ exist or does not exist?

Not well defined. But if the limit exist, we can defined $y = 3/4$ at $x=0$ so The function is now well defined and continuous at x=

What are the three basic types of discontinuities that we have studied? Give an example of each type.

Oscillatory. Jump. Unbounded.

Is the function $f(x) = \frac{1}{x}$ continuous at x=0? at x = 1 ? Explain why or why not. Explanation should involve a limit.

Not continuous at $x=0$. $f(x)$ is not even defined at $x=0$.

It is continuous at $x=1$. $\lim_{x \to 1^-} \frac{1}{x} = 1$ and $\lim_{x \to 1^+} \frac{1}{x} = 1$.

Is the absolute value function a continuous function at x=0 ? at x = 6 ? Explain why or why not. Explanation should involve a limit.

The absolute value function is continuous everywhere. There are no jumps or unbounded regions on its graph. The graph is a continuous joining of two lines.

Using the definition of the derivative, what is the derivative of x^2 with respect to x?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = 2x$$

Using the definition of the derivative, what is the derivative of $\frac{1}{x}$ with respect to x?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = -\frac{1}{x^2}$$

Using the definition of the derivative, explain why the absolute value function does not have a derivative at $x = 0$?

$$\lim_{x\to 0^-} \frac{d}{dx}\left(|x|\right) = -1 \qquad \lim_{x\to 0^+} \frac{d}{dx}\left(|x|\right) = 1$$

$-1 \neq 1$ so the derivative of $|x|$ is not continuous at $x=0$

List the six trigonometric functions and their derivatives.

$\sin x \to \cos x$ $\csc x \to -\csc x \cot x$

$\cos x \to -\sin x$ $\sec x \to \sec x \tan x$

$\tan x \to \sec^2 x$ $\cot x \to -\csc^2 x$

What are the derivatives with respect to x of the following trigonometric functions: $\sin(3x)$, $\cos(5x)$, $\tan(7x)$ and $\csc(8x)$.

$3\cos 3x \qquad -5\sin 5x \qquad 7\sec^2 7x \qquad -8\csc 8x \cot 8x$

What are the second derivatives with respect to t of the following trigonometric functions: $\sin(4t)$, $\cos(15t)$, and $\tan(9t)$.

$\sin 4t \to 4\cos 4t \to -16\sin 4t$

$\cos 15t \to -15\sin 15t \to -225\cos 15t$

$\tan 9t \to 9\sec^2 9t \to 9(2\cdot\sec 9t)(9\sec 9t \tan 9t)$

$= 162\sec^2 9t \tan 9t$

$f(x)$ $f'(x)$ $f''(x)$

What are the anti-derivatives with respect to t of the following trigonometric functions: $\sin(4t)$, $\cos(15t)$, and $\sec^2(9t)$.

$\boxed{-\frac{1}{4}\cos 4t + C}$ $\boxed{\frac{1}{15}\sin 15t + C}$ $\boxed{\frac{1}{9}\tan 9t + C}$

Evaluate: $\frac{d}{dx}\cos^3(\tan^3(4x))$

Chain Rule :

$$3\cos^2(\tan^3(4x))(-\sin(\tan^3(4x)))(3\tan^2(4x))(4\sec^2 4x)$$

Evaluate: $\frac{d}{dx}x^3\sin(17x-3)$

Product Rule

$$3x^2\sin(17x-3) + 17x^3\cos(17x-3)$$

Evaluate: $\frac{d}{dx}(\frac{x^3}{\sin(17x-3)})$

Division Rule

$$\frac{3x^2\sin(17x-3) - 17x^3\cos(17x-3)}{\sin^2(17x-3)}$$

Find $\frac{dy}{dx}$ if $xy - x^3y^2 - sin(xy) = 81$

Implicit

$$y + xy' - (3x^2y^2 + 2x^3yy') - \cos(xy)[y + xy'] = 0$$

Solve for y'

Find $\frac{dV}{dt}$ if $V = \frac{4}{3}\pi r^3$ if r is a function of t.

Related Rate

$$\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt} = 4\pi r^2\frac{dr}{dt}$$

Find $\frac{dV}{dt}$ if $V = \frac{1}{3}\pi r^2 h$ if r and h are functions of t.

Related Rate - Product Rule

$$\frac{dV}{dt} = \frac{dV}{dr}\cdot\frac{dr}{dt} + \frac{dV}{dh}\frac{dh}{dt} = \frac{2}{3}\pi r h\frac{dr}{dt} + \frac{1}{3}\pi r^2\frac{dh}{dt}$$

If a baseball is thrown straight up in the air at 96 feet per second, how high will it go? How long will it stay in the air?

$a = -32$

$v = -32t + v_0 = -32t + 96$

$s = -16t^2 + 96t + s_0 = -16t^2 + 96t$

Highest point when $v=0$ so $t=3$. $s(3) = -16\cdot 9 + 96\cdot 3 = 144$

In the air when $s=0$. $-16t^2 + 96t = t(96-16t)$, so $t=6$

If a baseball is dropped from a 480 feet tall building, how fast is it going when it hits the ground?

~~$a = -32$~~

~~$v = -32t$~~

~~$s = -32t$~~

$a = -32$

$v = -32t$

$s = -16t^2 + 480$

$s = 0$ when $-16t^2 + 480 = 0$

$t^2 = 30$

$t = \sqrt{30}$

So $v = -32\sqrt{30}$ when it hits the ground

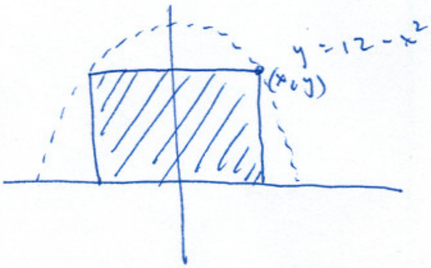
Find the smallest value of the function $y = 4x + \frac{6}{x}$.

min/max

$y' = 4 - \frac{6}{x^2}$ $\quad$ $y' = 0$ when $4 - \frac{6}{x^2} = 0$

$x^2 = \frac{6}{4}$ $\quad$ $x = \frac{\sqrt{6}}{2}$ $\quad$ $y = 4\frac{\sqrt{6}}{2} + \frac{6}{\frac{\sqrt{6}}{2}} = 4\sqrt{6}$

Find the dimensions of the largest rectangle that can fit in the region bounded by the parabola $y = 12 - x^2$ and $y = 0$.



Area of Rectangle $= 2xy$

$= 2x(12 - x^2)$

$= 24x - 2x^3$

Objective Function is Area

maximize when $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 24 - 6x^2$

$24 - 6x^2 = 0$

$6x^2 = 24$

$x^2 = 4$

$x = 2$ $\quad$ $y = 8$

So Rectangle has dimensions (4×8) with area 32.